Error Correction/Detection Decoding Scheme of Binary Hamming Codes

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SUMMARY An error correction/detection decoding scheme of binary Hamming codes is proposed. Error correction is performed by algebraic decoding and then error detection is performed by simple likelihood ratio testing. The proposed scheme reduces the probability of undetected decoding error in comparison with conventional error correction scheme and increases throughput in comparison with conventional error detection scheme.

key words: Binary Hamming codes, error correction/detection decoding scheme, algebraic decoding, likelihood ratio testing

1. Introduction

Error control coding is used for performance improvement in digital communications. Receiving incorrect data is worse than receiving no data at all in some applications. For those applications, error detection is more important than error correction.

Binary Hamming codes are perfect and single error correction codes [1]. With algebraic decoding, they can be used for either error correction only or error detection only. Therefore, it is required to use likelihood ratio for error correction/detection decoding schemes of binary Hamming codes.

Forney gives an optimal solution for error correction/detection schemes [2]. Forney’s optimal scheme requires high implementation complexity. Forney also proposes generalized minimum distance (GMD) decoding which is practically implemented [3]. In GMD decoding, the likelihood ratio is modified for the implementation of decoding algorithm but this modification causes performance degradation.

In this paper, an error correction/detection decoding scheme of binary Hamming codes is proposed. In Sect. 2, an error correction/detection decoding scheme is explained and compared with Forney’s optimal scheme. Simulation results are given in Sect. 3 and conclusion is drawn in Sect. 4.

2. Error Correction/Detection Decoding Scheme

Consider a Hamming code with code length \( n \) and dimension \( k \). Assume that coherent BPSK is used over an additive white gaussian noise (AWGN) channel. Let \( c \) be a codeword. The decision variable for \( j \)th code bit is given by

\[
r_j = (-1)^{c_j} \sqrt{2} E_o N_0 + e_j, \quad j = 1, 2, \ldots, n, \tag{1}
\]

where \( c_j \) is code bit, \( E_o / N_0 \) is bit energy to noise ratio, and \( e_j \) is a Gaussian random variable with zero mean and unit variance. Random variables \( e_j \)'s are mutually independent. Assume that \( |r_1| \leq |r_2| \leq \cdots \leq |r_n| \), i.e., the first bit is the least reliable and the \( n \)th bit is the most reliable among \( n \) bits. Let \( z_j \) be a hard decision value of \( r_j \). In a conventional error correction decoding scheme of binary Hamming codes, algebraic decoding always outputs an estimated codeword \( \hat{c} \) from \( z \). There is no error detection. In a conventional error detection decoding scheme, error is detected when \( z \) is not a codeword.

An error correction/detection decoding scheme is proposed. First, algebraic decoding is performed and then likelihood ratio testing is performed. For likelihood ratio testing, define a vector \( a \) of which component bit is given by

\[
a_j = \begin{cases} 1 - z_j, & j = 1, 2, \\ z_j, & j = 3, 4, \ldots, n. \end{cases} \tag{2}
\]

The estimated codeword \( \hat{c} \) from algebraic decoding is accepted, if the following inequality is satisfied:

\[
\frac{P(r | \hat{c})}{P(r | a)} \geq e^{nT}, \tag{3}
\]

where \( T \) is a likelihood ratio test parameter. The denominator of left-hand side is the likelihood that the vector \( z \) has two errors in the least and the second least reliable bits, i.e., in the first and second bits under our assumption. Taking logarithms on both sides and using the memoryless property of AWGN channel, (3) is written by

\[
\sum_{j=1}^{n} \log \frac{P(r_j | \hat{c}_j)}{P(r_j | a_j)} \geq nT. \tag{4}
\]

Let \( \lambda \) be a likelihood ratio for \( j \)th bit which is defined by

\[
\lambda_j = \log \frac{P(r_j | \hat{c}_j)}{P(r_j | 1 - z_j)}, \quad j = 1, 2, \ldots, n. \tag{5}
\]

The likelihood ratio \( \lambda_j \) is greater than zero or equal to...
zero since $z_j$ is a hard decision value of $r_j$. In left-hand side of (4), $j$th term has the following value: $\lambda_j$ when $\tilde{e}_j = z_j$ and $a_j = 1 - z_j - \lambda_j$ when $\tilde{e}_j = 1 - z_j$ and $a_j = z_j$, and zero when $\tilde{e}_j = a_j$. The first and second bits are different between $a$ and $z$. Let $e$ be the location of estimated error. The eth bit is different between $\tilde{e}$ and $z$. Therefore, (4) is written by

$$\lambda_1 + \lambda_2 - \lambda_e \geq nT. \tag{6}$$

Forney gives an optimal solution for error correction/detection decoding schemes. Forney's optimal scheme is that the estimated codeword $\tilde{e}$ is accepted, if the following inequality is satisfied:

$$\frac{P(\{r|\tilde{e}\})}{\sum_{\tilde{e}} P(\{r|\tilde{e}\})} \geq e^{nT}. \tag{7}$$

It is difficult to implement Forney's optimal scheme since it requires the computation of every codeword's likelihood.

The probability of undetected decoding error among accepted codewords is defined by

$$P_e = \frac{P_{ue}}{1 - P_{de}} \tag{8}$$

where $P_{ue}$ is the probability of undetected decoding error and $P_{de}$ is the probability of error detection. Both $P_{ue}$ and $P_{de}$ are achieved by computer simulation. The normalized throughput is defined by

$$\eta = \frac{k}{n(1 - P_{de})}. \tag{9}$$

3. Simulation Results

The probability of undetected decoding error among accepted codewords $P_e$ is shown in Fig. 1 for a (7, 4) Hamming code with the following decoding schemes: error correction, error detection, Forney's optimal error correction/detection, and proposed error correction/detection decoding schemes. In the proposed scheme and Forney's optimal scheme, we consider the case that the likelihood ratio test parameter $T$ is zero. The proposed scheme has 1.0 dB gain in the required $E_b/N_0$ for $P_e = 10^{-3}$ in comparison with error correction scheme though it has 0.57 dB loss in comparison with Forney's optimal scheme.

The normalized throughput is shown in Fig. 2. The proposed scheme has higher throughput in comparison with error detection scheme and has almost equal throughput in comparison with Forney's optimal scheme.

4. Conclusions

In this paper, we have proposed an error correction/detection decoding scheme of binary Hamming codes.
single algebraic decoding and simple likelihood ratio testing.

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References

