Outage Probability for Cooperative NOMA Systems With Imperfect SIC in Cognitive Radio Networks

Gyeongrae Im, *Student Member, IEEE*, and Jae Hong Lee, *Fellow, IEEE*

***Abstract*— In this letter, a cooperative non-orthogonal multiple access system with imperfect successive interference cancellation is investigated in an underlay cognitive radio network. Considering that the channel coefficients between the primary source and the secondary receiving nodes follow Rayleigh distribution, we derive an exact closed form of the exact outage probability for each secondary destination. Also, asymptotic expressions for the outage probability are derived: 1) when the interference constraint goes to infinity and 2) when the transmit powers at the secondary source and relay go to infinity. The simulation results verify our analytical results.**

***Index Terms*— NOMA, underlay cognitive radio, decode-and-forward, outage probability, imperfect SIC.**

I. Introduction

NON-ORTHGONAL multiple access (NOMA) has been emerged as a promising solution to achieve higher spectral efficiency in the fifth generation wireless networks [1]. Thanks to technical improvements of superposition coding and successive interference cancellation (SIC) in NOMA, multiple users can be served in the same resource block.

Cooperative NOMA is proposed in [2] to improve the reception reliability. The performance of cooperative NOMA is investigated in [3] and [4]. The achievable rate of the cooperative NOMA is investigated in [3]. Considering imperfect SIC, the ergodic sum capacity of the cooperative NOMA is investigated where two sources communicate with their corresponding destinations via a common relay [4].

Cooperative NOMA is applied in a cognitive radio network to enhance the spectral efficiency [5]. Approximated expressions for the outage probability are derived for a cooperative NOMA in an underlay cognitive radio network where one secondary user is selected as a relay [6]. In [7], the outage probability and the ergodic capacity of cooperative NOMA in an underlay cognitive radio network are derived assuming that the interference from the primary source is considered as a constant, which is not realistic in practice.

In this letter, we consider a cooperative NOMA in an underlay cognitive radio network. The main contributions of this letter are summarized as follows.

- We derive an exact closed form of the outage probability for each secondary destination under the assumption that the channel coefficients between the primary source to the secondary receiving nodes follow Rayleigh distribution.
- We consider the imperfect SIC scenario [4]. To our best knowledge, the cooperative NOMA with imperfect SIC in an underlay cognitive radio network has not been investigated yet.

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The authors are with the Department of Electrical and Computer Engineering, INMC, Seoul National University, Seoul 151-742, South Korea (e-mail: imgrae@snu.ac.kr; jhlee@snu.ac.kr).

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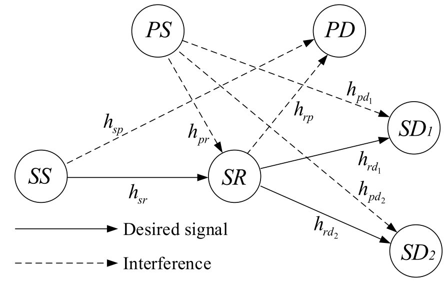

Fig. 1. System model. $h_{i,j}$ $(i \neq j,\ i \in \{p, s, r\},\ j \in \{p, r, d_1, d_2\})$ is the channel coefficient.

- We derive asymptotic expressions for the outage probability 1) when the interference constraint goes to infinity and 2) when the transmit powers at the secondary source and relay go to infinity. We remark that the diversity order of 0 is achieved which results in error floor at high interference constraint or high transmit power region. Simulation results show the validity of our analytical results.

II. System Model

Consider a cooperative NOMA system in an underlay cognitive radio network consisting of a primary source PS, a primary destination PD, a secondary source SS, a secondary decode-and-forward relay SR, and two secondary destinations SD_1, SD_2 as shown in Fig. 1. Assume that there is no direct path between SS and the secondary destinations. Suppose that each node has a single antenna and operates in half duplex mode. SS transmits a signal to the secondary destinations with the help of SR according to NOMA principle. A transmission frame of the network consists of two equal length phases. In the first phase, SS transmits a signal to SR. In the second phase, SR transmits the re-encoded signal to the secondary destinations only if the received signal at SR is decoded successfully. Assume that SD_2 has transmission priority over SD_1 [8]. Hence, SS and SR allocate more transmit power to the information symbol for SD_2 than that for SD_1. Assume imperfect SIC at SR and SD_1, i.e., some residual interference remains after SIC.

Assume that the channel coefficient is denoted as $h_{i,j}, i \neq j$, $i \in \{p, s, r\}$, $j \in \{p, r, d_1, d_2\}$, as shown in Fig. 1. $h_{i,j}$ is modeled as an independent zero-mean circularly symmetric complex Gaussian random variable with variance $\lambda_{i,j}^2$. Assume that $h_{i,j}$ is constant during a transmission frame and varies from one frame to another. Assume that all channels have an additive white Gaussian noise (AWGN) with zero mean and variance N_0.

In order to guarantee the quality of service requirement for PD, interference power at PD must be kept below a tolerable interference constraint Q.[1] The transmit power of SS and SR are limited by $P_s \leq \min\{P_s^{\rm m}, Q/|h_{sp}|^2\}$ and

[1] Q is determined to meet the target outage constraint of the primary destination [9]. If the outage constraint is small, interference from the secondary network should be limited to guarantee the performance of primary network which results in the performance degradation of the secondary network.

$$\Delta_1 = (1-e^{-\frac{Q}{P_s^m\lambda_{sp}}})e^{-\frac{N_0\kappa_1}{P_s^m\lambda_{sr}}}(\frac{P_s^m\lambda_{sr}}{P_s^m\lambda_{sr}+P_p\kappa_1\lambda_{pr}} - \frac{P_s^m\xi^2\alpha_2^2\kappa_2^2\lambda_{sr}}{(P_s^m\xi\alpha_2\kappa_2\lambda_{sr}+P_p\lambda_{pr}\varpi)(1+\xi\alpha_2\kappa_2)}e^{-\frac{N_0(\kappa_1-\kappa_2)}{P_s^m\xi\alpha_2\kappa_2\lambda_{sr}}}) \\ -\frac{Q\lambda_{sr}}{P_p\kappa_1\lambda_{sp}\lambda_{pr}}e^{-(\frac{1}{\lambda_{sp}}+\frac{N_0\kappa_1}{Q\lambda_{sr}})\frac{Q}{P_s^m}}\Xi((\frac{Q\lambda_{sr}+N_0\kappa_1\lambda_{sp}}{P_p\kappa_1\lambda_{sp}})(\frac{1}{\lambda_{pr}}+\frac{P_p\kappa_1}{P_s^m\lambda_{sr}})) + \frac{1}{\lambda_{sp}\lambda_{pr}}(1-\frac{1}{1+\xi\alpha_2\kappa_2}e^{-\frac{N_0(\kappa_1-\kappa_2)}{P_s^m\lambda_{sr}}}) \\ \times e^{-\frac{Q}{P_s^m\lambda_{sp}}-\frac{N_0\varpi}{P_s^m\xi\alpha_2\kappa_2\lambda_{sr}}}\frac{Q\xi\alpha_2\kappa_2\lambda_{sr}}{P_p\varpi}\Xi((\frac{Q\xi\alpha_2\kappa_2\lambda_{sr}+\lambda_{sp}N_0\varpi}{P_p\lambda_{sp}\varpi})(\frac{P_p\lambda_{pr}\varpi+P_s^m\lambda_{sr}\xi\alpha_2\kappa_2}{P_s^m\xi\lambda_{sr}\lambda_{pr}\alpha_2\kappa_2})) \quad (10)$$

$P_r \le \min\{P_r^{\mathrm{m}}, Q/|h_{rp}|^2\}$, respectively, where P_s^m and P_r^m are the maximum available power of SS and SR respectively.

In the first phase, the received signal at SR is given by

$$y_r = \sqrt{P_s}h_{sr}(\sqrt{\alpha_1}x_1+\sqrt{\alpha_2}x_2)+\sqrt{P_p}h_{pr}x_p+n_r \quad (1)$$

where P_p is the transmit power of PS, x_p is the information symbol for PD, x_k, $k\in\{1,2\}$, is the information symbol for SD_k, α_k is the power allocation coefficient for x_k with $\alpha_1+\alpha_2=1$ and $\alpha_1<\alpha_2$, and n_r is the AWGN at SR.

The signal to interference plus noise ratio (SINR) to decode x_2 at SR is given by

$$\Gamma_{x_2\to r} = \frac{P_s\alpha_2|h_{sr}|^2}{P_s\alpha_1|h_{sr}|^2+P_p|h_{pr}|^2+N_0}. \quad (2)$$

After imperfect SIC, the SINR to decode x_1 is given by

$$\Gamma_{x_1\to r} = \frac{P_s\alpha_1|h_{sr}|^2}{P_s\alpha_2|g_{sr}|^2+P_p|h_{pr}|^2+N_0} \quad (3)$$

where $g_{sr}\sim CN(0,\xi\lambda_{sr})$ and ξ, $0\le\xi\le1$, is the level of residual interference caused by imperfect SIC.[2]

In the second phase, SR transmits the signal consisting of the decoded and re-encoded symbols to the secondary destinations. The received signal at SD_k is given by

$$y_{d_k} = \sqrt{P_r}h_{rd_k}(\sqrt{\alpha_1}x_1+\sqrt{\alpha_2}x_2)+\sqrt{P_p}h_{pd_k}x_p+n_{d_k} \quad (4)$$

where n_{d_k} is the AWGN at SD_k, $k\in\{1,2\}$.

The SINR to decode x_2 at SD_1 is given by

$$\Gamma_{x_2\to d_1} = \frac{P_r\alpha_2|h_{rd_1}|^2}{P_r\alpha_1|h_{rd_1}|^2+P_p|h_{pd_1}|^2+N_0}. \quad (5)$$

After imperfect SIC, the SINR to decode x_1 is given by

$$\Gamma_{x_1\to d_1} = \frac{P_r\alpha_1|h_{rd_1}|^2}{P_r\alpha_2|g_{rd_1}|^2+P_p|h_{pd_1}|^2+N_0} \quad (6)$$

where $g_{rd_1}\sim CN(0,\xi\lambda_{rd_1})$.

The SINR to decode x_2 at SD_2 is given by

$$\Gamma_{x_2\to d_2} = \frac{P_r\alpha_2|h_{rd_2}|^2}{P_r\alpha_1|h_{rd_2}|^2+P_p|h_{pd_2}|^2+N_0}. \quad (7)$$

III. Outage Probability Analysis

A. Outage Probability at SD_1

An outage occurs at SD_1 in two cases. The first case is that SR fails to decode either x_1 or x_2. The second case is that SD_1 fails to decode either x_1 or x_2 even if SR successfully decode both x_1 and x_2.

[2]We use the imperfect SIC model in [4], where the imperfect SIC error obeys Gaussian distribution. However, there may be an error which does not follow Gaussian distribution in some complex cases [10]. These cases are beyond our scope, but should be considered in our future work.

The outage probability of the first case and the second case are given by

$$P_{out}^1 = 1-\underbrace{\Pr\{\Gamma_{x_2\to r}>\gamma_{x_2},\ \Gamma_{x_1\to r}>\gamma_{x_1}\}}_{\Delta_1} \quad (8)$$

and

$$P_{out}^2 = (1-P_{out}^1)(1-\underbrace{\Pr\{\Gamma_{x_2\to d_1}>\gamma_{x_2},\Gamma_{x_1\to d_1}>\gamma_{x_1}\}}_{\Delta_2}), \quad (9)$$

respectively, where $\gamma_k = 2^{2R_k}-1$ and R_k is the target rate to decode x_k, $k\in\{1,2\}$.

Theorem 1: For $\kappa_1>\kappa_2$, Δ_1 is given by (10) at the top of this page and for $\kappa_1\le\kappa_2$, Δ_1 is given by

$$\Delta_1 = (1-e^{-\frac{Q}{P_s^m\lambda_{sp}}})e^{-(\frac{N_0\kappa_2}{P_s^m\lambda_{sr}})}(\frac{1}{1+\xi\alpha_2\kappa_2}) \\ \times\{\frac{P_s^m\lambda_{sr}}{P_s^m\lambda_{sr}+P_p\lambda_{pr}\kappa_2} - \frac{Q\lambda_{sr}}{P_p\lambda_{sp}\lambda_{pr}\kappa_2}e^{-\frac{Q}{P_s^m\lambda_{sp}}-\frac{N_0\kappa_2}{P_s^m\lambda_{sr}}} \\ \times\Xi(\frac{Q\lambda_{sr}+N_0\lambda_{sp}\kappa_2}{P_p\lambda_{sp}\kappa_2}\frac{P_s^m\lambda_{sr}+P_p\lambda_{pr}\kappa_2}{P_s^m\lambda_{sr}\lambda_{pr}0})\} \quad (11)$$

where $\Xi(x) = e^xEi(-x)$, $Ei(x) = -\int_{-x}^{\infty}\frac{e^{-t}}{t}\,dt$ is the exponential integral function, $\kappa_1 = \frac{\gamma_{x_2}}{(\alpha_2-\gamma_{x_2}\alpha_1)}$, $\kappa_2 = \frac{\gamma_{x_1}}{\alpha_1}$, and $\varpi = \kappa_1-\kappa_2+\xi\kappa_1\kappa_2\alpha_2$.

Proof: From (2), (3), and (8), Δ_1 is given by

$$\Delta_1 = \Pr\{|h_{sp}|^2<\frac{Q}{P_s^m}, |h_{sr}|^2>\frac{\kappa_1}{P_s^m}(P_p|h_{pr}|^2+N_0), \\ |h_{sr}|^2>\alpha_2\kappa_2|g_{sr}|^2+\frac{\kappa_2}{P_s^m}(P_p|h_{pr}|^2+N_0)\} \\ +\Pr\{|h_{sp}|^2>\frac{Q}{P_s^m}, \frac{|h_{sr}|^2}{|h_{sp}|^2}>\frac{\kappa_1}{Q}(P_p|h_{pr}|^2+N_0), \\ \frac{|h_{sr}|^2}{|h_{sp}|^2}>\frac{\alpha_2\kappa_2|g_{sr}|^2}{|h_{sp}|^2}+\frac{\kappa_2}{Q}(P_p|h_{pr}|^2+N_0)\}. \quad (12)$$

For $\kappa_1>\kappa_2$, (12) becomes

$$\Delta_1 = \int_0^\infty\int_0^{\frac{Q}{P_s^m}}\int_{\frac{a_1(z)}{P_s^m}}^\infty\int_0^{\frac{a_2(z)}{P_s^m}} f(x,y,z,w)dwdydxdz \\ +\int_0^\infty\int_0^{\frac{Q}{P_s^m}}\int_{a_3(z,w)}^\infty\int_{\frac{a_2(z)}{P_s^m}}^\infty f(x,y,z,w)dwdydxdz \\ +\int_0^\infty\int_{\frac{Q}{P_s^m}}^\infty\int_0^{\frac{xa_2(z)}{Q}}\int_{\frac{xa_1(z)}{Q}}^\infty f(x,y,z,w)dydwdxdz \\ +\int_0^\infty\int_{\frac{Q}{P_s^m}}^\infty\int_{\frac{xa_2(z)}{Q}}^\infty\int_{a_4(x,z,w)}^\infty f(x,y,z,w)dydwdxdz \quad (13)$$

and for $\kappa_1 \le \kappa_2$, (12) becomes

$$\Delta_1 = \int_0^\infty \int_0^{\frac{Q}{P_s^m}} \int_0^\infty \int_{a_3(z,w)}^\infty f(x,y,z,w)dydwdxdz + \int_0^\infty \int_0^{\frac{Q}{P_s^m}} \int_0^\infty \int_{a_4(x,z,w)}^\infty f(x,y,z,w)dydwdxdz \quad (14)$$

where $a_1(z) = (P_p z + N_0)\kappa_1$, $a_2(z) = \frac{P_p(\kappa_1-\kappa_2)}{\alpha_2\kappa_2}z + \frac{N_0(\kappa_1-\kappa_2)}{\alpha_2\kappa_2}$, $a_3(z,w) = \alpha_2\kappa_2 w + \frac{\kappa_2}{P_s^m}(P_p z + N_0)$, $a_4(x,z,w) = \alpha_2\kappa_2 w + \frac{\kappa_2}{Q}x(P_p z + N_0)$, and $f(x,y,z,w) = \frac{1}{\xi\lambda_{sp}\lambda_{sr}^2\lambda_{pr}} \times e^{-\frac{1}{\lambda_{sp}}x - \frac{1}{\lambda_{sr}}y - \frac{1}{\lambda_{pr}}z - \frac{1}{\xi\lambda_{sr}}w}$ is the joint probability density function (PDF) of $|h_{sp}|^2$, $|h_{sr}|^2$, $|h_{pr}|^2$, and $|g_{sr}|^2$. By simple mathematical manipulation, we obtain (10) and (11) using the fact that $\int_0^\infty \frac{e^{-\mu x}}{x+\beta}dx = -e^{\beta\mu}Ei(-\beta\mu)$ [11, eq. (3.352.4)]. ∎

From (8), (10), and (11), the outage probability of the first case can be obtained in closed form.

Because Δ_2 in (9) has a similar form to Δ_1 in (8), the closed form of Δ_2 can be obtained by using the same approach.

The outage probability at SD_1 is given by

$$P_{out}^{d_1} = P_{out}^1 + P_{out}^2 = 1 - \Delta_1\Delta_2. \quad (15)$$

B. Outage Probability at SD_2

An outage occurs at SD_2 in two cases. The first case is that SR fails to decode either x_1 or x_2, the same case as in the previous subsection. The second case is that SD_2 fails to decode its desired symbol x_2 even if SR successfully decodes both x_1 and x_2.

The outage probability of the second case is given by

$$P_{out}^3 = (1 - P_{out}^1)\underbrace{\Pr[\Gamma_{x_2\to d_2} < \gamma_{x_2}]}_{\Delta_3}. \quad (16)$$

Theorem 2: The closed form of Δ_3 in (16) is given by

$$\Delta_3 = 1 - (1 - e^{-\frac{Q}{P_r^m\lambda_{rp}}})e^{-\frac{N_0\kappa_1}{P_r^m\lambda_{rd_2}}}\frac{P_r^m\lambda_{rd_2}}{P_r^m\lambda_{rd_2} + P_p\kappa_1\lambda_{pd_2}} + \frac{Q\lambda_{rd_2}}{P_p\kappa_1\lambda_{rp}\lambda_{pd_2}}e^{-(\frac{1}{\lambda_{rp}} + \frac{N_0\kappa_1}{Q\lambda_{rd_2}})\frac{Q}{P_r^m}} \times\Xi((\frac{Q\lambda_{rd_2} + N_0\kappa_1\lambda_{rp}}{b\lambda_{rp}})(\frac{1}{\lambda_{pd_2}} + \frac{P_p\kappa_1}{P_r^m\lambda_{rd_2}})). \quad (17)$$

Proof: By using (7), Δ_3 is given by

$$\begin{aligned}\Delta_3 &= \Pr\{|h_{rp}|^2 < \frac{Q}{P_r^m}, |h_{rd_2}|^2 < \frac{P_p\kappa_1|h_{pd_2}|^2}{P_r^m} + \frac{N_0\kappa_1}{P_r^m}\} \\ &\quad + \Pr\{|h_{rp}|^2 > \frac{Q}{P_r^m}, \frac{|h_{rd_2}|^2}{|h_{rp}|^2} < \frac{P_p\kappa_1|h_{pd_2}|^2}{Q} + \frac{N_0\kappa_1}{Q}\} \\ &= \int_0^\infty \int_0^{\frac{Q}{P_r^m}} \int_0^{\frac{P_p\kappa_1}{P_r^m}z + \frac{N_0\kappa_1}{P_r^m}} g(x,y,z)dydxdz \\ &\quad + \int_0^\infty \int_{\frac{Q}{P_r^m}}^\infty \int_0^{\frac{P_p\kappa_1}{Q}xz + \frac{N_0\kappa_1}{Q}x} g(x,y,z)dydxdz \end{aligned} \quad (18)$$

where $g(x,y,z) = \frac{1}{\lambda_{rp}\lambda_{rd_2}\lambda_{pd_2}} e^{-\frac{1}{\lambda_{rp}}x - \frac{1}{\lambda_{rd_2}}y - \frac{1}{\lambda_{pd_2}}z}$ is the joint PDF of $|h_{rp}|^2$, $|h_{rd_2}|^2$, and $|h_{pd_2}|^2$. By simple manipulation from (18), closed form of Δ_3 can be obtained. ∎

The outage probability at SD_2 is given by

$$P_{out}^{d_2} = P_{out}^1 + P_{out}^3 = 1 - \Delta_1 + \Delta_1\Delta_3. \quad (19)$$

IV. Asymptotic Expressions

Because derived closed expressions are intractable, we analyze the asymptotic expressions for the outage probability to provide additional insight.

A. Asymptotic Expressions With $Q \to \infty$

Corollary 1: When the interference constraint Q goes to infinity, the asymptotic expression for Δ_1 is as follows:

For $\kappa_1 > \kappa_2$, the asymptotic expression for Δ_1 is given by

$$\Delta_1^Q = e^{-\frac{N_0\kappa_1}{P_s^m\lambda_{sr}}}\{\frac{P_s^m\lambda_{sr}}{P_s^m\lambda_{sr} + P_p\kappa_1\lambda_{pr}} - e^{-\frac{N_0(\kappa_1-\kappa_2)}{P_s^m\xi\alpha_2\lambda_{sr}\kappa_2}} \times\frac{P_s^m\xi^2\alpha_2^2\kappa_2^2\lambda_{sr}}{(1+\xi\alpha_2\kappa_2)(P_s^m\xi\alpha_2\kappa_2\lambda_{sr} + P_p\lambda_{pr}\varpi)}\} \quad (20)$$

and for $\kappa_1 \le \kappa_2$,

$$\Delta_1^Q = \frac{1}{1+\xi\alpha_2\kappa_2}\frac{P_s^m\lambda_{sr}}{P_s^m\lambda_{sr} + P_p\kappa_2\lambda_{pr}}e^{-\frac{N_0\kappa_2}{P_s^m\lambda_{sr}}}. \quad (21)$$

Proof: For $x > 0$, following inequality holds [12].

$$\frac{1}{2}\ln(1 + \frac{2}{x}) < e^x E_1(x) < \ln(1 + \frac{1}{x}). \quad (22)$$

Since $\lim_{x\to\infty} \frac{x}{a}\ln(1 + \frac{a}{x}) = 1$ for arbitrary real number $a \ne 0$ and $E_1(x) = -E_i(-x)$, $-xe^x E_i(-x)$ approaches 1 as x goes to ∞, i.e.,

$$\lim_{x\to\infty} -xe^x E_i(-x) = 1. \quad (23)$$

From (10), (11), and (23), $Corollary$ 1 can be proved straightforwardly. ∎

The asymptotic expression for Δ_2, Δ_2^Q, can be obtained similarly. Then the asymptotic expression for $P_{out}^{d_1}$ is given by

$$\lim_{Q\to\infty} P_{out}^{d_1} = 1 - \Delta_1^Q\Delta_2^Q. \quad (24)$$

Corollary 2: From (17) and (23), the asymptotic expression for Δ_3 when Q goes to infinity is given by

$$\Delta_3^Q = 1 - e^{-\frac{N_0\kappa_1}{P_r^m\lambda_{rd_2}}}\frac{P_r^m\lambda_{rd_2}}{P_r^m\lambda_{rd_2} + P_p\kappa_1\lambda_{pd_2}}. \quad (25)$$

The asymptotic expression for $P_{out}^{d_2}$ when Q goes to infinity is given by

$$\lim_{Q\to\infty} P_{out}^{d_2} = 1 - \Delta_1^Q + \Delta_1^Q\Delta_3^Q. \quad (26)$$

B. Asymptotic Expressions With $P_s^m \to \infty$, $P_r^m \to \infty$

Corollary 3: When P_s^m goes to infinity, the asymptotic expression for Δ_1 are as follows:
For $\kappa_1 > \kappa_2$,

$$\Delta_1^{P_s^m} = -\frac{Q\lambda_{sr}}{P_p\kappa_1\lambda_{sp}\lambda_{pr}}\Xi(\frac{Q\lambda_{sr} + N_0\kappa_1\lambda_{sp}}{P_p\kappa_1\lambda_{pr}\lambda_{sp}}) + \frac{Q\xi^2\alpha_2\kappa_2{}^2\lambda_{sr}}{P_p\lambda_{sp}\lambda_{pr}\varpi(1+\xi\alpha_2\kappa_2)}\Xi(\frac{Q\xi\alpha_2\kappa_2\lambda_{sr} + N_0\lambda_{sp}\varpi}{P_p\lambda_{pr}\lambda_{sp}\varpi}) \quad (27)$$

and for $\kappa_1 \le \kappa_2$,

$$\Delta_1^{P_s^m} = -\frac{Q\lambda_{sr}}{(1+\xi\alpha_2\kappa_2)P_p\kappa_2\lambda_{sp}\lambda_{pr}}\Xi(\frac{Q\lambda_{sr} + N_0\kappa_2\lambda_{sp}}{P_p\kappa_2\lambda_{pr}\lambda_{sp}}). \quad (28)$$

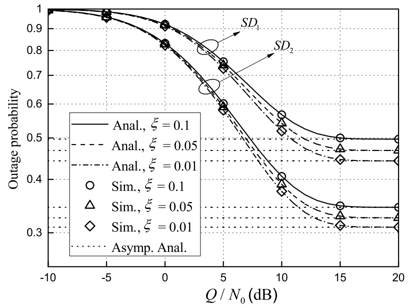

Fig. 2. Outage probability versus the interference constraint. $P_s^m/N_0 = 10$dB.

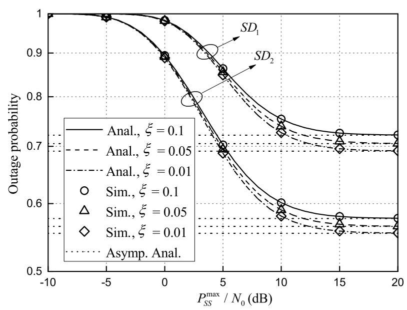

Fig. 3. Outage probability versus the maximum available transmit power of secondary source. $Q/N_0 = 10$dB.

Proof: When $P_s^m \to \infty$, Q/P_s^m goes to 0. Plugging $Q/P_s^m = 0$ into (10) and (11), we have (27) and (28), respectively. ■

The asymptotic expression for Δ_2, $\Delta_2^{P_r^m}$, can be obtained similarly. The asymptotic expression for $P_{out}^{d_1}$ is given by

$$\lim_{P_s^m, P_r^m \to \infty} P_{out}^{d_1} = 1 - \Delta_1^{P_s^m} \Delta_2^{P_r^m}. \tag{29}$$

Corollary 4: When P_r^m goes to infinity, the asymptotic expression for Δ_3 is given by

$$\Delta_3^{P_r^m} = \frac{Q\lambda_{rd_2}}{P_p\kappa_1\lambda_{pd_2}\lambda_{rp}} \Xi\left(\frac{Q\lambda_{rd_2} + N_0\kappa_1\lambda_{rp}}{P_p\kappa_1\lambda_{pd_2}\lambda_{rp}}\right). \tag{30}$$

The asymptotic expression for $P_{out}^{d_2}$ is given by

$$\lim_{P_s^m, P_r^m \to \infty} P_{out}^{d_2} = 1 - \Delta_1^{P_s^m} + \Delta_1^{P_s^m} \Delta_3^{P_r^m}. \tag{31}$$

Remark. When Q goes to infinity, the transmit power of SS and that of SR are P_s^m and P_r^m respectively, with probability 1. Therefore, the asymptotic expressions in (24) and (26) are not related to Q. From the definition of the diversity order which is defined as $d = -\lim_{Q\to\infty} \log P_{out}^{d_i}(Q)/\log(Q)$, the diversity order of 0 is achieved. Likewise, when $P_s^m \to \infty$ and $P_r^m \to \infty$, the diversity order of 0 is achieved. We can expect that there exist an error floor at the high interference constraint or high transmit power region, which is a similar result with [13].

V. Simulation Results and Conclusion

Assume that $\lambda_{i,j}^2 = 1$ for $i \neq j$, $i \in \{p, s, r\}$, $j \in \{p, r, d_1, d_2\}$. Suppose that $\alpha_1 = 0.2$, $\alpha_2 = 0.8$, and $P_s^m = P_r^m$.

Fig. 2 shows the outage probability versus Q/N_0 for $P_s^m/N_0 = 10$dB. It is shown that the outage probability decreases as Q/N_0 increases. It is also shown that the outage probability increases as ξ increases. As expected, error floors occur in the high interference constraint region of which the values are exactly equal to our analytical results in (24) and (26) for all plots in Fig. 2. Also, the value of the error floor increases as ξ increases.

Fig. 3 shows the outage probability versus P_s^m/N_0 for $Q/N_0 =$10dB. It is shown that the outage probability increases as P_s^m/N_0 increases. It is also shown that the outage probability increases as ξ increases. For all plots in Fig. 3, error floors occur in the high maximum available transmit power region of which the values are exactly equal to our analytical results in (29) and (31). Also, the value of the error floor increases as ξ increases.

In Fig. 2 and Fig. 3, it is shown that the analysis for outage probability perfectly matches with the simulation results.

In conclusion, we investigate the outage probability of a cooperative NOMA system with imperfect successive interference cancellation in an underlay cognitive radio network. Considering interference from the primary source and interference constraint to the primary destination, we derive the outage probability of each secondary destination and its asymptotic expressions exactly. It is shown that our analytical results match perfectly with simulation results.

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