Outage Probability of Two-Way Full-Duplex Relaying With Imperfect Channel State Information

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***Abstract*—In this letter, we investigate two-way full-duplex (TWFD) relaying with a residual loop interference (LI). In the TWFD relaying, two full-duplex users exchange data with each other via a full-duplex relay, and each node attempts to subtract the estimate of the residual LI from its received signal. We derive the exact integral and approximate closed-form expressions for the outage probability of the TWFD relaying in case of perfect and imperfect channel state information. Monte Carlo simulations verify the validity of analytical results.**

***Index Terms*—Full-duplex, two-way relaying, outage probability, imperfect channel state information (CSI).**

I. INTRODUCTION

RELAYING is an effective way to combat the performance degradation caused by fading, shadowing, and path loss [1], [2]. Two-way relaying, where two users exchange information with each other via a single or multiple relays, provides improved spectral efficiency compared to conventional one-way relaying by using either superposition coding or physical layer network coding at relays [3], [4].

A full-duplex scheme, where the transmission and the reception occur at the same time on the same channel, achieves up to double the capacity of a half-duplex scheme [5]–[8]. Although the full-duplex scheme suffers from loop interference (LI), it has drawn attention due to recent advances on interference cancellation and transmit/receive antenna isolation to mitigate the LI [9]–[11].

Relaying and full-duplex schemes are combined together to achieve higher data rates [6], [12]–[15]. In [6], the authors investigate one-way full-duplex (OWFD) relaying and two-way half-duplex (TWHD) relaying in order to minimize/recover the spectral efficiency loss associated with owe-way half-duplex (OWHD) relaying which requires additional resources (e.g. time slots or frequencies) to transmit data. In [12] and [13], the authors present OWFD relaying with multiple antennas in order to provide a solution to overcome the spectral efficiency loss in OWHD relaying. In [14], the authors investigate OWFD relaying with opportunistic relay selection in order to enhance the performance of OWHD relaying. However, most previous works are focused on OWFD relaying, and there have been few works on two-way full-duplex (TWFD) relaying. In [15], the authors study TWFD relaying with power allocation to maximize the average rate. However, they ignore the residual LI at each node, which is one of the important factors that should be considered in the performance analysis of the full-duplex scheme. They leave it as an interesting open problem.

In this letter, we investigate TWFD relaying in the presence of residual LI. The performance of the TWFD relaying with LI is analyzed in case of perfect and imperfect channel state information (CSI). Analytical results are verified by Monte Carlo simulations.

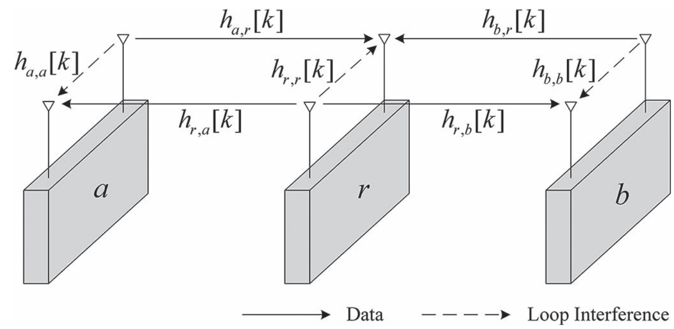

Fig. 1. System model for two-way full-duplex relaying.

II. SYSTEM MODEL

Consider TWFD relaying, where the users a and b exchange information with each other via an amplify-and-forward (AF) relay r as shown in Fig. 1. Assume that each of the users a, b, and the relay r has a transmit antenna and a receive antenna, and there is no direct path between the users a and b. Assume that the relay r is located at the middle between the users a and b.

All the channels are assumed to be block fading, i.e., the channel remains constant over a block but changes independently from one block to another. Assume that the channel from the node i to the node j at the block k has the channel coefficient $h_{i,j}[k]$, i, $j \in \{a, b, r\}$, which is a zero-mean circularly symmetric complex Gaussian (CSCG) random variable with the variance $\sigma^2_{h_{i,j}}$.[1] We model the channel $h_{i,j}[k]$ as the sum of the channel estimate $\hat{h}_{i,j}[k]$ and the channel estimation error $\Delta h_{i,j}[k]$, i.e., [15]

$$h_{i,j}[k] = \hat{h}_{i,j}[k] + \Delta h_{i,j}[k]. \quad (1)$$

[1]We model the channel coefficient of LI as a zero-mean CSCG random variable under the assumptions that the line-of-sight component is effectively reduced by the antenna isolation/interference cancelation but the scattering multi-path components still remain due to imperfect cancellation [10], [14]. Assume that the channels are not reciprocal, i.e., $h_{i,j}[k] \neq h_{j,i}[k]$, since there are two different links, i.e., (i node's transmit antenna, j node's receive antenna) link, (j node's transmit antenna, i node's receive antenna) link.

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Assume that the channel estimate $\hat{h}_{i,j}[k]$ and the channel estimation error $\Delta h_{i,j}[k]$ are mutually uncorrelated, which is valid for minimum mean-square error estimation. The channel estimate $\hat{h}_{i,j}[k]$ and the channel estimation error $\Delta h_{i,j}[k]$ are zero-mean CSCG random variables with the variances $\sigma^2_{\hat{h}_{i,j}}$ and $\sigma^2_{\Delta h_{i,j}} = \sigma^2_{h_{i,j}} - \sigma^2_{\hat{h}_{i,j}}$, respectively.

Assume that the users a and b communicate with each other via the AF relay r. At the block k, the users a and b transmit their signals to the relay r simultaneously, and, at the same time, the relay r broadcasts its signal to the users a and b. Then, the relay r receives not only the transmitted signals from the users a and b but also the LI from the relay r itself. Then, the received signal at the relay r is given by

$$y_r[k] = \sqrt{E_a} h_{a,r}[k] x_a[k] + \sqrt{E_b} h_{b,r}[k] x_b[k] + \sqrt{E_r} h_{r,r}[k] x_r[k] + n_r[k] \quad (2)$$

where $x_i[k]$ is the transmit signal from the node i with unit power, E_i is the transmit energy from the node i, $i \in \{a, b, r\}$, $n_r[k]$ is the additive white Gaussian noise (AWGN) with zero mean and variance N_0 at the relay r, and the third term in the right hand side is the LI from the relay r itself. The relay r subtracts an estimate of the LI from its received signal which yields

$$\tilde{y}_r[k] = y_r[k] - \sqrt{E_r} \hat{h}_{r,r}[k] x_r[k]. \quad (3)$$

From (1), (2), and (3), we have

$$\begin{aligned}\tilde{y}_r[k] &= \sqrt{E_a} \hat{h}_{a,r}[k] x_a[k] + \sqrt{E_b} \hat{h}_{b,r}[k] x_b[k] \\ &\quad + \sqrt{E_a} \Delta h_{a,r}[k] x_a[k] + \sqrt{E_b} \Delta h_{b,r}[k] x_b[k] \\ &\quad + \sqrt{E_r} \Delta h_{r,r}[k] x_r[k] + n_r[k]. \end{aligned} \quad (4)$$

The transmit signal from the relay r is given by $x_r[k] = \alpha[k-1] \tilde{y}_r[k-1]$ where the amplification factor is given by

$$\alpha[k-1] = \left(E_a \left|\hat{h}_{a,r}[k-1]\right|^2 + E_b \left|\hat{h}_{b,r}[k-1]\right|^2 + E_a \sigma^2_{\Delta h_{a,r}} + E_b \sigma^2_{\Delta h_{b,r}} + E_r \sigma^2_{\Delta h_{r,r}} + N_0 \right)^{-\frac{1}{2}}. \quad (5)$$

The received signal at the user a is given by[2]

$$y_a[k] = \sqrt{E_r} h_{r,a}[k] x_r[k] + \sqrt{E_a} h_{a,a}[k] x_a[k] + n_a[k] \quad (6)$$

where $n_a[k]$ is the AWGN with zero mean and variance N_0 at the user a. The user a subtracts the estimates of the self-interference (SI)[3] and the LI from its received signal which yields (7), shown at the bottom of the page.

By dividing the desired signal power by the interference and noise power, the signal-to-interference-plus-noise ratio (SINR) at the user a is given by

$$\begin{aligned}\gamma_a =& E_r E_b \left|\hat{h}_{r,a}[k]\right|^2 \left|\hat{h}_{b,r}[k-1]\right|^2 \\ &\times \Big(E_r E_b \sigma^2_{\Delta h_{r,a}} \left|\hat{h}_{b,r}[k-1]\right|^2 + E_r E_a \sigma^2_{\Delta h_{r,a}} \left|\hat{h}_{a,r}[k-1]\right|^2 \\ &\quad + E_r E_a \left|\hat{h}_{r,a}[k]\right|^2 \sigma^2_{\Delta h_{a,r}} + E_r E_a \sigma^2_{\Delta h_{r,a}} \sigma^2_{\Delta h_{a,r}} \\ &\quad + E_r E_b \left|\hat{h}_{r,a}[k]\right|^2 \sigma^2_{\Delta h_{b,r}} + E_r E_b \sigma^2_{\Delta h_{r,a}} \sigma^2_{\Delta h_{b,r}} \\ &\quad + E_r^2 \left|\hat{h}_{r,a}[k]\right|^2 \sigma^2_{\Delta h_{r,r}} + E_r^2 \sigma^2_{\Delta h_{r,a}} \sigma^2_{\Delta h_{r,r}} \\ &\quad + E_r \left|\hat{h}_{r,a}[k]\right|^2 N_0 + E_r \sigma^2_{\Delta h_{r,a}} N_0 \\ &\quad + |\alpha[k-1]|^{-2} E_a \sigma^2_{\Delta h_{a,a}} + |\alpha[k-1]|^{-2} N_0 \Big)^{-1} \\ =& \frac{\chi_1 \left|\hat{h}_{r,a}[k]\right|^2 \chi_2 \left|\hat{h}_{b,r}[k-1]\right|^2}{\chi_1 \left|\hat{h}_{r,a}[k]\right|^2 + \chi_2 \left|\hat{h}_{b,r}[k-1]\right|^2 + \chi_3 \left|\hat{h}_{a,r}[k-1]\right|^2 + \chi_4} \end{aligned} \quad (8)$$

where

$$\chi_1 = \frac{E_r}{E_r \sigma^2_{\Delta h_{r,a}} + E_a \sigma^2_{\Delta h_{a,a}} + N_0} \quad (9)$$

$$\chi_2 = \frac{E_b}{E_a \sigma^2_{\Delta h_{a,r}} + E_b \sigma^2_{\Delta h_{b,r}} + E_r \sigma^2_{\Delta h_{r,r}} + N_0} \quad (10)$$

$$\chi_3 = \frac{E_a}{E_a \sigma^2_{\Delta h_{a,r}} + E_b \sigma^2_{\Delta h_{b,r}} + E_r \sigma^2_{\Delta h_{r,r}} + N_0} \quad (11)$$

[2]In the following, we will focus on the user a since similar approach and analysis can be applied for the user b.

[3]For the SI cancellation, each node has to obtain the necessary CSI. At the beginning of each block (e.g., k), the CSI acquisition is achieved in four steps. In the first step, the user a transmits its pilot signal to the relay r, and the relay r estimates $h_{a,r}[k]$. In the second step, the user b transmits its pilot signal to the relay r, and the relay r estimates $h_{b,r}[k]$. In the third step, the relay r broadcasts the pilot signal, and the users a and b estimate $h_{r,a}[k]$ and $h_{r,b}[k]$, respectively. In the fourth step, the relay r feeds back $\hat{h}_{a,r}[k-1]$, $\hat{h}_{b,r}[k-1]$, and $\alpha[k-1]$ to the users a and b. In order to reduce the feedback overhead, the relay r can feed back their quantized version to the users a and b [16].

$$\begin{aligned}\tilde{y}_a[k] =& y_a[k] - \alpha[k-1]\sqrt{E_r E_a} \hat{h}_{r,a}[k] \hat{h}_{a,r}[k-1] x_a[k-1] - \sqrt{E_a} \hat{h}_{a,a}[k] x_a[k] \\ =& \alpha[k-1]\sqrt{E_r E_b} \hat{h}_{r,a}[k] \hat{h}_{b,r}[k-1] x_b[k-1] + \alpha[k-1]\sqrt{E_r E_b} \Delta h_{r,a}[k] \hat{h}_{b,r}[k-1] x_b[k-1] \\ &+ \alpha[k-1]\sqrt{E_r E_a} \Delta h_{r,a}[k] \hat{h}_{a,r}[k-1] x_a[k-1] + \alpha[k-1]\sqrt{E_r E_a} \hat{h}_{r,a}[k] \Delta h_{a,r}[k-1] x_a[k-1] \\ &+ \alpha[k-1]\sqrt{E_r E_a} \Delta h_{r,a}[k] \Delta h_{a,r}[k-1] x_a[k-1] + \alpha[k-1]\sqrt{E_r E_b} \hat{h}_{r,a}[k] \Delta h_{b,r}[k-1] x_b[k-1] \\ &+ \alpha[k-1]\sqrt{E_r E_b} \Delta h_{r,a}[k] \Delta h_{b,r}[k-1] x_b[k-1] + \alpha[k-1]\sqrt{E_r E_r} \hat{h}_{r,a}[k] \Delta h_{r,r}[k-1] x_r[k-1] \\ &+ \alpha[k-1]\sqrt{E_r E_r} \Delta h_{r,a}[k] \Delta h_{r,r}[k-1] x_r[k-1] + \alpha[k-1]\sqrt{E_r} \hat{h}_{r,a}[k] n_r[k-1] + \alpha[k-1]\sqrt{E_r} \Delta h_{r,a}[k] n_r[k-1] \\ &+ \sqrt{E_a} \Delta h_{a,a}[k] x_a[k] + n_a[k] \end{aligned} \quad (7)$$

$$\begin{aligned}\chi_4 =& \Big(E_r E_a \sigma^2_{\Delta h_{r,a}} \sigma^2_{\Delta h_{a,r}} + E_r E_b \sigma^2_{\Delta h_{r,r}} \sigma^2_{\Delta h_{b,r}} \\ &+ E_r E_r \sigma^2_{\Delta h_{r,a}} \sigma^2_{\Delta h_{r,r}} + E_r \sigma^2_{\Delta h_{r,a}} N_0 \\ &+ E_a E_a \sigma^2_{\Delta h_{a,a}} \sigma^2_{\Delta h_{a,r}} + E_b E_a \sigma^2_{\Delta h_{a,a}} \sigma^2_{\Delta h_{b,r}} \\ &+ E_r E_a \sigma^2_{\Delta h_{a,a}} \sigma^2_{\Delta h_{r,r}} + E_a \sigma^2_{\Delta h_{a,a}} N_0 + E_a \sigma^2_{\Delta h_{a,r}} N_0 \\ &+ E_b \sigma^2_{\Delta h_{b,r}} N_0 + E_r \sigma^2_{\Delta h_{r,r}} N_0 + N_0 N_0\Big) \\ &\times \Big(E_r \sigma^2_{\Delta h_{r,a}} + E_a \sigma^2_{\Delta h_{a,a}} + N_0\Big)^{-1} \\ &\times \Big(E_a \sigma^2_{\Delta h_{a,r}} + E_b \sigma^2_{\Delta h_{b,r}} + E_r \sigma^2_{\Delta h_{r,r}} + N_0\Big)^{-1}. \end{aligned} \quad (12)$$

Note that for perfect channel estimation, i.e., when $\sigma^2_{\Delta h_{i,j}} = 0$ for $i, j \in \{a, b, r\}$, (9)–(12) reduce to $\chi_1 = E_r/N_0$, $\chi_2 = E_b/N_0$, $\chi_3 = E_a/N_0$, and $\chi_4 = 1$.

III. Outage Probability

The cumulative distribution function (CDF) of the SINR at the user a is given by

$$\begin{aligned} F_{\gamma_a}(\gamma) =& \Pr\left(\frac{\chi_1 |\hat{h}_{r,a}|^2 \chi_2 |\hat{h}_{b,r}|^2}{\chi_1 |\hat{h}_{r,a}|^2 + \chi_2 |\hat{h}_{b,r}|^2 + \chi_3 |\hat{h}_{a,r}|^2 + \chi_4} < \gamma\right) \\ =& \int_0^\infty \int_0^{\frac{\gamma}{\chi_2}} \Pr\left(|\hat{h}_{r,a}|^2 > \frac{\gamma\chi_3 y + \gamma\chi_2 x + \gamma\chi_4}{\chi_1(\chi_2 x - \gamma)}\right) \\ &\times f_{|\hat{h}_{b,r}|^2}(x) f_{|\hat{h}_{a,r}|^2}(y)\, dx\, dy \\ &+ \int_0^\infty \int_{\frac{\gamma}{\chi_2}}^\infty \Pr\left(|\hat{h}_{r,a}|^2 < \frac{\gamma\chi_3 y + \gamma\chi_2 x + \gamma\chi_4}{\chi_1(\chi_2 x - \gamma)}\right) \\ &\times f_{|\hat{h}_{b,r}|^2}(x) f_{|\hat{h}_{a,r}|^2}(y)\, dx\, dy \end{aligned} \quad (13)$$

where, for notational simplicity, the block indices k and $k-1$ in $\hat{h}_{r,a}[k]$, $\hat{h}_{a,r}[k-1]$, and $\hat{h}_{b,r}[k-1]$ are omitted. Since $|\hat{h}_{r,a}|^2$, $|\hat{h}_{a,r}|^2$, and $|\hat{h}_{b,r}|^2$ are independent, exponentially distributed random variables with parameters $1/\sigma^2_{\hat{h}_{r,a}}$, $1/\sigma^2_{\hat{h}_{a,r}}$, and $1/\sigma^2_{\hat{h}_{b,r}}$, respectively, we have

$$F_{\gamma_a}(\gamma) = 1 - \int_0^\infty \int_{\frac{\gamma}{\chi_2}}^\infty \frac{e^{-\frac{x}{\sigma^2_{\hat{h}_{b,r}}} - \frac{y}{\sigma^2_{\hat{h}_{a,r}}} - \frac{\gamma\chi_3 y + \gamma\chi_2 x + \gamma\chi_4}{\sigma^2_{\hat{h}_{r,a}} \chi_1(\chi_2 x - \gamma)}}}{\sigma^2_{\hat{h}_{a,r}} \sigma^2_{\hat{h}_{b,r}}}\, dx\, dy. \quad (14)$$

Putting $x = (1/\chi_1\chi_2)(z + \chi_1\gamma)$ into (14), we get

$$\begin{aligned} & F_{\gamma_a}(\gamma) \\ &= 1 - \frac{e^{-\frac{\gamma}{\sigma^2_{\hat{h}_{r,a}}\chi_1} - \frac{\gamma}{\sigma^2_{\hat{h}_{b,r}}\chi_2}}}{\sigma^2_{\hat{h}_{a,r}} \sigma^2_{\hat{h}_{b,r}} \chi_1\chi_2} \int_0^\infty e^{-\frac{y}{\sigma^2_{\hat{h}_{a,r}}}} \int_0^\infty e^{-\frac{\gamma\chi_3 y + \gamma^2 + \gamma\chi_4}{\sigma^2_{\hat{h}_{r,a}} z}} \\ &\quad \times e^{-\frac{z}{\sigma^2_{\hat{h}_{b,r}}\chi_1\chi_2}}\, dz\, dy \\ &\overset{(a)}{=} 1 - 2e^{-\frac{\gamma}{\sigma^2_{\hat{h}_{r,a}}\chi_1} - \frac{\gamma}{\sigma^2_{\hat{h}_{b,r}}\chi_2}} \int_0^\infty e^{-\frac{y}{\sigma^2_{\hat{h}_{a,r}}}} \sqrt{\frac{\gamma\chi_3 y + \gamma^2 + \gamma\chi_4}{\sigma^4_{\hat{h}_{a,r}} \sigma^2_{\hat{h}_{b,r}} \sigma^2_{\hat{h}_{r,a}} \chi_1\chi_2}} \\ &\quad \times K_1\left(2\sqrt{\frac{\gamma\chi_3 y + \gamma^2 + \gamma\chi_4}{\sigma^2_{\hat{h}_{r,a}} \sigma^2_{\hat{h}_{b,r}} \chi_1\chi_2}}\right) dy \\ &\overset{(b)}{=} 1 - e^{-\frac{\gamma}{\sigma^2_{\hat{h}_{r,a}}\chi_1} - \frac{\gamma}{\sigma^2_{\hat{h}_{b,r}}\chi_2} + \frac{\gamma + \chi_4}{\sigma^2_{\hat{h}_{a,r}}\chi_3}} \\ &\quad \times \Bigg\{\psi_1\gamma e^{\psi_1\gamma}\Gamma(-1, \psi_1\gamma) - \frac{1}{4\psi_1\gamma} \underbrace{\int_0^{\psi_2} e^{-\frac{y}{4\psi_1\gamma}} \sqrt{y} K_1(\sqrt{y})\, dy}_{\triangleq\, \Xi(\gamma)}\Bigg\} \end{aligned} \quad (15)$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind in [17, eq. (8.407.1)], $\psi_1 = \sigma^2_{\hat{h}_{a,r}}\chi_3/(\sigma^2_{\hat{h}_{b,r}}\sigma^2_{\hat{h}_{r,a}}\chi_1\chi_2)$, $\psi_2 = (4\gamma^2 + 4\gamma\chi_4)/(\sigma^2_{\hat{h}_{b,r}}\sigma^2_{\hat{h}_{r,a}}\chi_1\chi_2)$, and $\Gamma(\cdot,\cdot)$ is the incomplete gamma function in [17, eq. (8.350.2)]. In (15), (a) follows from the fact that $\int_0^\infty \exp(-(p/x) - qx)\, dx = 2\sqrt{p/q}K_1(2\sqrt{pq})$ in [17, eq. (3.471.9)]; (b) follows from the fact that $\int_0^\infty \exp(-\lambda x)\sqrt{\kappa x + \eta} K_1(\mu\sqrt{\kappa x + \eta}) dx = (\kappa\mu)/(4\lambda^2)\exp((\kappa\mu^2/4\lambda) + (\eta\lambda/\kappa))\,\Gamma(-1, (\kappa\mu^2/4\lambda)) - (1/\kappa)\exp(\eta\lambda/\kappa)\int_0^\eta \exp(-(\lambda x/\kappa))\sqrt{x}K_1(\mu\sqrt{x})\, dx$ in [18, vol. 4, eq. (1.1.2.3)] and [18, vol. 4, eq. (3.16.2.4)]. The evaluation of the integral in (15) is difficult due to the product of the first order modified Bessel function of the second kind and the exponential function. We thus make an approximation.

By the M-th order Taylor series approximation of the exponential function $e^{-x} \simeq \sum_{m=0}^{M}(-x^m/m!)$, the integral in (15) is given by

$$\begin{aligned} \Xi(\gamma) &\simeq \sum_{m=0}^{M}\left(-\frac{1}{4\psi_1\gamma}\right)^m \int_0^{\psi_2} \frac{(y)^{m+\frac{1}{2}}}{m!} K_1(\sqrt{y})\, dy \\ &\overset{(c)}{=} \sum_{m=0}^{M}\left(-\frac{1}{4\psi_1\gamma}\right)^m \int_0^{\psi_2} \frac{(y)^{m+\frac{1}{2}}}{2m!} G^{2\,0}_{0\,2}\left(\frac{y}{4}\,\middle|\,\frac{1}{2}, -\frac{1}{2}\right) dy \\ &\overset{(d)}{=} \sum_{m=0}^{M}\left(-\frac{1}{4\psi_1\gamma}\right)^m \frac{1}{2m!}\,\psi_2^{m+\frac{3}{2}} \\ &\quad \times G^{21}_{13}\left(\frac{\psi_2}{4}\,\middle|\,\begin{matrix} -m-\frac{1}{2} \\ \frac{1}{2}, -\frac{1}{2}, -m-\frac{3}{2}\end{matrix}\right) \end{aligned} \quad (16)$$

where $G^{pq}_{vu}(\cdot)$ is the Meijer's G-function in [17, eq. (9.301)]. In (16), (c) follows from the fact that $K_1(\sqrt{y}) = (1/2)G^{2\,0}_{0\,2}((y/4)|(1/2), -(1/2))$ in [17, eq. (9.34.3)] and (d) follows from the fact that $\int_0^x y^{m+(1/2)} G^{2\,0}_{0\,2}((y/4)|(1/2), -(1/2))dy = x^{m+(3/2)}\, G^{21}_{13}\left((x/4)\,|\,\begin{smallmatrix}-m-(3/2) \\ (1/2), -(1/2), -m-(1/2)\end{smallmatrix}\right)$ in [18, vol. 3, eq. (1.16.2.1)].

From (15) and (16), the CDF of the SINR at the user a is given by

$$\begin{aligned} F_{\gamma_a}(\gamma) \simeq& 1 - e^{-\frac{\gamma}{\sigma^2_{\hat{h}_{r,a}}\chi_1} - \frac{\gamma}{\sigma^2_{\hat{h}_{b,r}}\chi_2} + \frac{\gamma + \chi_4}{\sigma^2_{\hat{h}_{a,r}}\chi_3}} \\ & \Bigg\{\psi_1\gamma e^{\psi_1\gamma}\Gamma(-1, \psi_1\gamma) + \sum_{m=0}^{M}\frac{1}{2m!}\left(-\frac{1}{4\psi_1\gamma}\right)^{m+1} \\ & \times \left(\frac{4\gamma^2 + 4\chi_4\gamma}{\sigma^2_{\hat{h}_{b,r}}\sigma^2_{\hat{h}_{r,a}}\chi_1\chi_2}\right)^{m+\frac{3}{2}} \\ & \times G^{21}_{13}\left(\frac{\gamma^2 + \chi_4\gamma}{\sigma^2_{\hat{h}_{b,r}}\sigma^2_{\hat{h}_{r,a}}\chi_1\chi_2}\,\middle|\,\begin{matrix} -m-\frac{1}{2} \\ \frac{1}{2}, -\frac{1}{2}, -m-\frac{3}{2}\end{matrix}\right)\Bigg\}. \end{aligned} \quad (17)$$

An outage occurs when the SINR at the user a falls below a SINR threshold γ_{th}. The outage probability of the user a is given by

$$P_{\text{out},a}(\gamma_{\text{th}}) = \Pr(\gamma_a \leq \gamma_{\text{th}}) = F_{\gamma_a}(\gamma_{\text{th}}). \quad (18)$$

In the high signal-to-noise ratio (SNR) regime ($E_a = E_b = E_r = E \to \infty$), the diversity order determines the slope of the outage probability of the user a versus E/N_0 [1], [2]. From (15) and (18), using $e^x \simeq 1 + x$, $\Gamma(-1, x)/x^{-1} \simeq 1$, and $K_1(x) \simeq 1/x$ as $x \to 0$, the outage probability of the user a is approximated as

$$\begin{aligned} P_{\text{out},a}(\gamma_{\text{th}}) \approx & \frac{\gamma_{\text{th}}}{\sigma^2_{\hat{h}_{r,a}} \chi_1} + \frac{\gamma_{\text{th}}}{\sigma^2_{\hat{h}_{b,r}} \chi_2} - \psi_1 \gamma_{\text{th}} \\ \approx & \frac{\gamma_{\text{th}} \left(E\sigma^2_{\Delta h_{r,a}} + E\sigma^2_{\Delta h_{a,a}} + N_0 \right)}{\sigma^2_{\hat{h}_{r,a}} E} \\ & + \frac{\gamma_{\text{th}} \left(E\sigma^2_{\Delta h_{a,r}} + E\sigma^2_{\Delta h_{b,r}} + E\sigma^2_{\Delta h_{r,r}} + N_0 \right)}{\sigma^2_{\hat{h}_{b,r}} E} \\ & - \frac{\sigma^2_{\hat{h}_{a,r}} \left(E\sigma^2_{\Delta h_{r,a}} + E\sigma^2_{\Delta h_{a,a}} + N_0 \right)}{\sigma^2_{\hat{h}_{b,r}} \sigma^2_{\hat{h}_{r,a}} E \gamma_{\text{th}}}. \end{aligned} \quad (19)$$

When perfect channel estimation is available ($\sigma^2_{\Delta h_{i,j}} = 0, i, j \in \{a, b, r\}$), (19) is rewritten as

$$P_{\text{out},a}(\gamma_{\text{th}}) \approx \left(\frac{\gamma_{\text{th}}}{\sigma^2_{\hat{h}_{r,a}}} + \frac{\gamma_{\text{th}}}{\sigma^2_{\hat{h}_{b,r}}} - \frac{\sigma^2_{\hat{h}_{a,r}}}{\sigma^2_{\hat{h}_{b,r}} \sigma^2_{\hat{h}_{r,a}}} \gamma_{\text{th}} \right) \left(\frac{E}{N_0} \right)^{-1} \quad (20)$$

which is a function of $(E/N_0)^{-1}$. Therefore, the diversity order is one. When perfect channel estimation is not available, we can expect that the error floor appears at high E/N_0 region.

IV. Numerical Results And Discussion

Consider TWFD relaying where users a and b exchange information with the help of an AF relay r. Suppose that the transmit energy at the users a, b, and the relay r is same, i.e., $E_a = E_b = E_r = E$. And the transmit SNR is defined as $\text{SNR} = E/N_0$ [2]. For simplicity, we will use the notation $(\sigma^2_{\Delta h_{a,r}}, \sigma^2_{\Delta h_{r,r}}, \sigma^2_{\Delta h_{b,r}}, \sigma^2_{\Delta h_{a,a}}, \sigma^2_{\Delta h_{r,a}})$ in the figure to denote the variances of the channel estimation errors.

Fig. 2 shows the outage probability of the user a versus SNR for the TWFD relaying when $\sigma^2_{h_{a,r}} = \sigma^2_{h_{b,r}} = \sigma^2_{h_{r,a}} = 1$, $\sigma^2_{h_{r,r}} = \sigma^2_{h_{a,a}} = 10$, and $\gamma_{\text{th}} = 1$ dB. The analytical results are generated based on (17) and (18). It is shown that the analytical results perfectly match the simulation results. In case 1, all variances of the channel estimation errors are set to 0. In cases 2–6, one of the variances of the channel estimation errors is set to 0.1. In case 7, all variances of the channel estimation errors are set to 0.1. It is shown that the outage probability of cases 2–4 is lower than that of cases 5–6. The reason is that, due to power normalization (5) at the relay, the effects of $\sigma^2_{\Delta h_{a,r}}$, $\sigma^2_{\Delta h_{r,r}}$, and $\sigma^2_{\Delta h_{b,r}}$ on the outage probability are reduced. It is shown that, as SNR increases, the difference in the outage probability between case 1 and cases 2–7 increases. The reason is that the outage probability of case 1 decreases as SNR increases and that of cases 2–7 exhibits error floors at high SNR region.

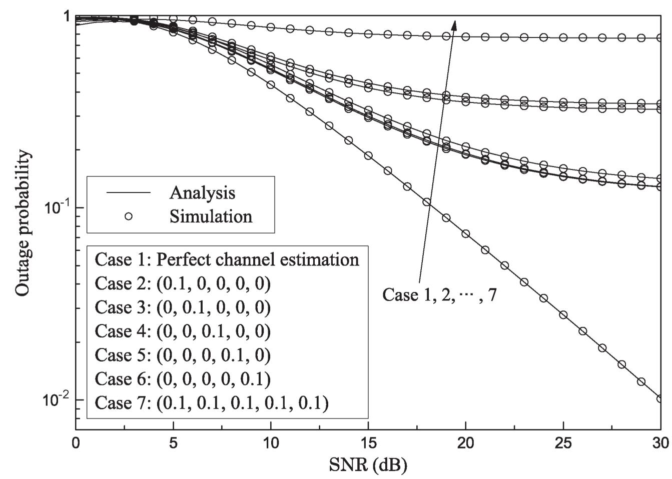

Fig. 2. Outage probability of the user a versus SNR for TWFD relaying.

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