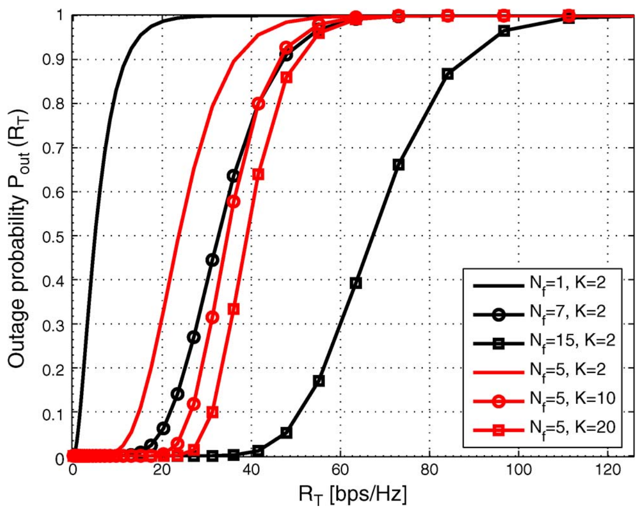

Fig. 5. Outage probability $P_{\mathrm{out}}(R_T)$ with $\rho = 6$ dB.

scheduling over the QRD-based CP SC transmissions, and two representative scenarios $K = 1$ and $N_f = 1$ have been studied. For arbitrary values of K and N_f, the derived closed-form expressions for the maximum average achievable rate have also been verified by the empirical simulations. The simulation results have shown the tightness of the derived closed-form expressions between exact and empirical ones. From the simulations, we can also verify that the derived outage diversity gain is determined by the number of active users and the length of channel taps.

REFERENCES

[1] F. Horlin and A. Bourdoux, *Digital Compensation for Analog Front-Ends: A New Approach To Wireless Tranceiver Design*. Hoboken, NJ: Wiley, 2008.

[2] D. Falconer, S. L. Ariyavisitakul, A. B. Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 58–66, Apr. 2002.

[3] Y. Zeng, Y. Liang, and C. Xu, "Semi-blind channel estimation for linearly precoded MIMO-CPSC," in *Proc. ICC*, Beijing, China, May 2008, pp. 604–608.

[4] *Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for High Rate Wireless Personal Area Networks (WPANs): Amendment 2: Millimeter-wave based Alternative Physical Layer Extension*, IEEE Std. P802.15.3c/D00, 2008.

[5] *Proposal for PAR and 5 Criteria for Very High Throughput (VHT) SG for 60 GHz*, IEEE Std. 802.11/11-08-0223-05, May 2008.

[6] P. Smulders, "Exploiting the 60 GHz band for local wireless multimedia access: Prospects and future directions," *IEEE Commun. Mag.*, vol. 40, no. 1, pp. 140–147, Jan. 2002.

[7] M.-O. Pun, K. J. Kim, R. A. Iltis, and H. V. Poor, "Reduced-feedback opportunistic scheduling and beamforming with GMD for MIMO-OFDMA," in *Proc. Asilomar Conf. Signals Syst. Comput.*, Monterey, CA, Oct. 2008, pp. 2009–2013.

[8] C.-J. Chen and L.-C. Wang, "Performance analysis of scheduling in multiuser MIMO systems wit zero-forcing receivers," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1435–1445, Sep. 2007.

[9] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE ICC*, Jun. 1995, vol. 1, pp. 331–335.

[10] H. Cho and J. G. Andrews, "Resource-redistributive opportunistic scheduling for wireless systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3510–3522, Jul. 2009.

[11] W. Zhang, "Comments on maximum diversity in single-carrier frequency-domain equalization," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 1275–1276, Mar. 2006.

[12] K. J. Kim, Y. Yue, R. A. Iltis, and J. D. Gibson, "A QRD-M/Kalman filter-based detection and channel estimation algorithm for MIMO-OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 710–721, Mar. 2005.

[13] M. Baek, Y. You, and H. Song, "Combined QRD-M and DFE detection technique for simple and efficient signal detection in MIMO-OFDM system," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1632–1638, Apr. 2009.

[14] G. H. Golub and C. F. Van Loan, *Matrix Computations*. Baltimore, MD: The Johns Hopkins Univ. Press, 1983.

[15] M. K. Simon, *Probability Distributions Involving Gaussian Random Variables*. New York: Springer-Verlag, 2006.

[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. New York: Academic, 2007.

[17] V. S. Adamchik and O. I. Marichev, "The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system," in *Proc. Int. Conf. Symbolic Algebraic Comput.*, Tokyo, Japan, 1990, pp. 212–224.

[18] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integral and Series*, 4th ed. London, U.K.: Gordon and Breach, 1998.

[19] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1972.

[20] M.-S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165–1181, Jul. 1999.

[21] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*. New York: Wiley, 2005.

Outage Probability for Dual-Hop Relaying Systems With Multiple Interferers Over Rayleigh Fading Channels

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***Abstract*—In this paper, we analyze the dual-hop amplify-and-forward (AF) and decode-and-forward (DF) relaying systems with multiple interferers over Rayleigh fading channels. We derive closed-form expressions for the outage probability of the dual-hop AF and DF relaying systems with multiple interferers, which have arbitrary transmit power. Numerical results verify the validity of our theoretical analysis by comparison with Monte Carlo simulations and compare the outage probabilities of the dual-hop AF and DF relaying systems with multiple interferers.**

***Index Terms*—Amplify and forward (AF), decode and forward (DF), dual-hop relaying, interference limited wireless networks, outage probability.**

I. INTRODUCTION

Dual-hop relaying is well known as an efficient way to extend coverage area and overcome channel impairments, such as fading, shadowing, and path loss, in wireless fading channel environments [1]–[4]. In dual-hop relaying, when the direct link between the source and the destination is deeply faded, a source communicates with a

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destination via an intermediate relay. If the relay just amplifies and forwards the source data to the destination, then it is called an amplify-and-forward (AF) relaying system. On the other hand, if the relay decodes the source data, reencodes, and forwards it to the destination, then it is called a decode-and-forward (DF) relaying system.

The performance analysis of the dual-hop AF and DF relaying systems over fading channels has extensively been studied. In [5] and [6], the end-to-end outage and error probability of the dual-hop AF and DF relaying systems were studied over Rayleigh and Nakagami-m fading channels. In [7], the authors presented and characterized four channel models for multihop relaying and introduced the concept of multihop diversity. The performance bounds of the multihop AF relaying system with channel state information (CSI)-assisted and fixed-gain relays were investigated over Nakagami-m fading [8]. However, most of the previous works on the dual-hop relaying systems have focused on wireless networks with no interference, and there have been few works on the dual-hop relaying systems in interference-limited wireless networks. In [6], the authors investigated the outage probability of the dual-hop AF relaying system with multiple interferers over Nakagami-m fading channels. However, the analytical result for the outage probability of the dual-hop AF relaying system has no closed-form expression but does have a complicated integral expression, and it is valid only for the case where all cochannel interferers have the same transmit power. In [9]–[11], the authors investigated the outage probability of the dual-hop AF and DF relaying systems over Rayleigh fading channels. However, to simplify the derivation of the outage probability, the authors considered a simplified interference model [9], [10] and assumed no interference at the relay [11]. In [12], the authors investigated the relay selection and the statistical behavior of the AF cooperative diversity in ad hoc networks with multiuser interference. However, it considered the interference model where the relays receive the interfering signal from a single interferer and the destination is interference free and assumed high signal-to-noise ratio approximation to simply derive the approximate outage probability.

In this paper, we investigate the dual-hop AF and DF relaying systems with multiple interferers over Rayleigh fading channels. We consider the interference model where both the relay and the destination receive the interfering signals from multiple cochannel interferers, which have arbitrary transmit power. We derive the closed-form expressions for the outage probability of the dual-hop AF and DF relaying systems with multiple interferers. Numerical results verify the validity of our theoretical analysis by comparison with Monte Carlo simulations and compare the outage probabilities of the dual-hop AF and DF relaying systems with multiple interferers.

This paper is organized as follows: In Section II, we describe the system model for the dual-hop AF and DF relaying systems with multiple interferers over Rayleigh fading channels. In Section III, we derive the closed-form expressions for the outage probability of the dual-hop AF and DF relaying systems with multiple interferers. In Section IV, the numerical results verify the validity of the performance analysis by comparison between the analytical results and Monte Carlo simulations. In addition, the outage probabilities of the dual-hop AF and DF relaying systems are compared. Finally, conclusions are drawn in Section V.

II. System Model

Consider the dual-hop relaying system with multiple interferers over Rayleigh fading channels, as shown in Fig. 1. Assume that a source T_0 communicates with a destination T_2 via a relay T_1. Assume that independent frequency-flat slow fading channels and CSI are only known at the receiving terminals. Assume that a communication between the source and the destination is performed in two phases under the half-duplex constraint. In the first phase, the source transmits the data to the relay, and then, the relay receives the source data and the interfering signals from I_1 interfering terminals. At the relay, the received signal from the source and I_1 interfering terminals is given by

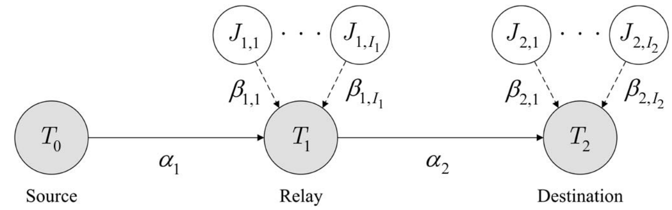

Fig. 1. System model.

$$y_1 = \alpha_1 x_{1,0} + \sum_{i=1}^{I_1} \beta_{1,i} x_{1,i} + n_1 \tag{1}$$

where α_k is the fading channel coefficient from the terminal T_{k-1} to the terminal T_k, $x_{k,0}$ is the transmitted signal from the terminal T_{k-1} to the terminal T_k, $\beta_{k,i}$ is the fading channel coefficient from the ith interfering terminal $J_{k,i}$ to the terminal T_k, $x_{k,i}$ is the interfering signal from the ith interfering terminal $J_{k,i}$ to the terminal T_k, and n_k is the additive noise from the terminal T_{k-1} to the terminal T_k. Assume that the fading channel coefficient α_k and $\beta_{k,i}$ and the noise n_k are independent zero-mean circularly symmetric complex Gaussian random variables with variance λ_k^2, $\mu_{k,i}^2$, and N_0, respectively. The channel variances $\lambda_{i,j}^2$ and $\mu_{i,j}^2$ are given by $\lambda_{i,j}^2 = \eta D_{i,j}^{-\nu}$ and $\mu_{i,j}^2 = \eta D_{i,j}^{-\nu}$, respectively, where η is the propagation constant, $D_{i,j}$ is the distance between the terminal i and the terminal j, and ν is the path loss exponent.

For the dual-hop AF relaying system, in the second phase, the relay amplifies the received signal y_1 and forwards it to the destination, and then, the destination receives the amplified signal from the relay and the interfering signals from I_2 interfering terminals. At the destination, the received signal from the relay and I_2 interfering terminals is given by

$$\begin{aligned} y_2^{AF} &= G\alpha_2 y_1 + \sum_{i=1}^{I_2} \beta_{2,i} x_{2,i} + n_2 \\ &= G\alpha_2\alpha_1 x_{1,0} + G\alpha_2 \sum_{i=1}^{I_1} \beta_{1,i} x_{1,i} + \sum_{i=1}^{I_2} \beta_{2,i} x_{2,i} \\ &\quad + G\alpha_2 n_1 + n_2 \end{aligned} \tag{2}$$

where G is the amplification factor at the relay. From [5], [6], and [8], the relay gain is chosen to be

$$G^2 = \frac{1}{|\alpha_1|^2} \tag{3}$$

where the relay just amplifies the received signal with the inverse of the channel between the source and the relay. As mentioned in [5] and [6], such a relay serves as a benchmark for all practical AF relaying systems. In addition, it makes the statistical analysis more tractable.

For the dual-hop DF relaying system, in the second phase, the relay decodes the source data $x_{1,0}$, reencodes, and forwards it to the destination, and then, the destination receives the reencoded source data $x_{2,0}$ and the interfering signals from I_2 interfering terminals. At the destination, the received signal from the relay and I_2 interfering terminals is given by

$$y_2^{\mathrm{DF}} = \alpha_2 x_{2,0} + \sum_{i=1}^{I_2} \beta_{2,i} x_{2,i} + n_2. \tag{4}$$

III. Outage Probability

A. AF Relaying System

For the dual-hop AF relaying system, from (1) and (2), the received signal-to-interference ratio (SIR) at the destination is given by

$$\begin{aligned}\gamma_2^{AF} &= \left[\frac{1}{P_0|\alpha_1|^2}\sum_{i=1}^{I_1} P_{1,i}|\beta_{1,i}|^2 + \frac{1}{P_0|\alpha_2|^2}\sum_{i=1}^{I_2} P_{2,i}|\beta_{2,i}|^2\right]^{-1}\\ &= \left[\frac{1}{X_1} + \frac{1}{X_2}\right]^{-1}\end{aligned} \tag{5}$$

where $P_0 = |x_{k,0}|^2$ is the transmit power per hop, $P_{k,i} = |x_{k,i}|^2$ is the transmit power of the interfering signals, and the random variable $X_k = P_0|\alpha_k|^2/\sum_{i=1}^{I_k} P_{k,i}|\beta_{k,i}|^2$ for $k = 1, 2$.

Theorem 1 (PDF of X_k): Let the channel gains $|\alpha_k|^2$ and $|\beta_{k,i}|^2$ be independent exponential random variables with hazard rates $1/\lambda_k$ and $1/\mu_{k,i}$, $k = 1, 2$ and $i = 1, \ldots, I_k$, respectively. Then, given a random variable $X_k = P_0|\alpha_k|^2/\sum_{i=1}^{I_k} P_{k,i}|\beta_{k,i}|^2$, the probability density function (pdf) of X_k is given by

$$f_{X_k}(x) = \sum_{i=1}^{\rho(\boldsymbol{\Omega}_k)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_k)} \chi_{i,j}(\boldsymbol{\Omega}_k) \frac{j\Omega_{k,[i]}}{\Omega_{k,0}} \left(1 + \frac{\Omega_{k,[i]}x}{\Omega_{k,0}}\right)^{-j-1} \tag{6}$$

where $\Omega_{k,0} = P_0\lambda_k$, $\Omega_{k,i} = P_{k,i}\mu_{k,i}$, $\boldsymbol{\Omega}_k = \mathrm{diag}(\Omega_{k,1}, \ldots, \Omega_{k,i}, \ldots, \Omega_{k,I_k})$, $\rho(\boldsymbol{\Omega}_k)$ denotes the number of distinct diagonal elements of $\boldsymbol{\Omega}_k$, $\Omega_{k,[1]} > \Omega_{k,[2]} > \cdots > \Omega_{k,[\rho(\boldsymbol{\Omega}_k)]}$ are the distinct diagonal elements in decreasing order, $\tau_i(\boldsymbol{\Omega}_k)$ is the multiplicity of $\Omega_{k,[i]}$, and $\chi_{i,j}(\boldsymbol{\Omega}_k)$ is the (i, j)th characteristic coefficient of $\boldsymbol{\Omega}_k$ [13, eq. (129)].

Proof: See Appendix A. ∎

For example, when all of $\Omega_{k,i}$ are equal, that is, $\Omega_k = \Omega_{k,i}$ for $i = 1, 2, \ldots, I_k$, then the pdf of X_k is rewritten as

$$f_{X_k}(x) = \frac{I_k\Omega_k}{\Omega_{k,0}}\left(1 + \frac{\Omega_k x}{\Omega_{k,0}}\right)^{-I_k-1}. \tag{7}$$

For another example, when all of $\Omega_{k,i}$ are distinct, that is, $\Omega_{k,i} \neq \Omega_{k,j}$ for $i \neq j$, then the pdf of X_k is rewritten as

$$f_{X_k}(x) = \sum_{i=1}^{I_k}\left\{\prod_{\substack{j=1\\ j\neq i}}^{I_k}\left(1 - \frac{\Omega_{k,j}}{\Omega_{k,i}}\right)^{-1}\right\}\frac{\Omega_{k,i}}{\Omega_{k,0} + \Omega_{k,i}x}. \tag{8}$$

Lemma 1 (PDF of $1/X_k$): With the help of [14, Ch. 5.2], the pdf of $Y_k = 1/X_k$ is given by

$$f_{Y_k}(y) = \frac{1}{y^2}\sum_{i=1}^{\rho(\boldsymbol{\Omega}_k)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_k)} j\chi_{i,j}(\boldsymbol{\Omega}_k)\left(\frac{\Omega_{k,0}}{\Omega_{k,[i]}}\right)^j \left(\frac{1}{y} + \frac{\Omega_{k,0}}{\Omega_{k,[i]}}\right)^{-j-1}. \tag{9}$$

Lemma 2 (MGF of $1/X_k$): From Lemma 1 and [15, eq. (1.2)], the moment generating function (MGF) of $Y_k = 1/X_k$ is given by

$$M_{Y_k}(s) = \sum_{i=1}^{\rho(\boldsymbol{\Omega}_k)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_k)} \chi_{i,j}(\boldsymbol{\Omega}_k) j\Gamma(j)\Psi\left(j, 0, \frac{s\Omega_{k,[i]}}{\Omega_{k,0}}\right) \tag{10}$$

where $\Gamma(j)$ is the Gamma function defined as $\Gamma(j) = (j-1)!$ for a positive integer j in [16, eq. (8.310.1)], and $\Psi(\alpha, \beta, z)$ is the confluent hypergeometric function of the second kind defined in [16, eq. (9.211.4)].

Theorem 2 (CDF of γ_2^{AF}): For the dual-hop AF relaying system with multiple interferers, the cumulative distribution function (cdf) of γ_2^{AF} is given by

$$\begin{aligned}F_{\gamma_2^{AF}}(\gamma) = 1 - &\sum_{i=1}^{\rho(\boldsymbol{\Omega}_1)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_1)} \sum_{l=1}^{\rho(\boldsymbol{\Omega}_2)} \sum_{m=1}^{\tau_l(\boldsymbol{\Omega}_2)} \chi_{i,j}(\boldsymbol{\Omega}_1)\chi_{l,m}(\boldsymbol{\Omega}_2)\\ &\times\left[\frac{jm\Gamma(j)\Gamma(m)}{\Gamma(j+m+1)}\left(1+\frac{\Omega_{1,[i]}\gamma}{\Omega_{1,0}}\right)^{-j}\left(1+\frac{\Omega_{2,[l]}\gamma}{\Omega_{2,0}}\right)^{-m}\right.\\ &\left.\times\, {}_2F_1\left(j, m; j+m+1; \kappa_1(\gamma)\right)\right]\end{aligned} \tag{11}$$

where ${}_2F_1(a, b; c; z)$ is Gauss' hypergeometric function defined in [17, eq. (15.1.1)], and

$$\kappa_\upsilon(\gamma) = \frac{\upsilon + \frac{\Omega_{1,[i]}\gamma}{\Omega_{1,0}} + \frac{\Omega_{2,[l]}\gamma}{\Omega_{2,0}}}{\left(1 + \frac{\Omega_{1,[i]}\gamma}{\Omega_{1,0}}\right)^\upsilon\left(1 + \frac{\Omega_{2,[l]}\gamma}{\Omega_{2,0}}\right)^\upsilon}. \tag{12}$$

Proof: See Appendix B. ∎

Corollary 1 (PDF of γ_2^{AF}): For the dual-hop AF relaying system with multiple interferers, the pdf of γ_2^{AF} is given by

$$\begin{aligned}f_{\gamma_2^{AF}}(\gamma) = &\sum_{i=1}^{\rho(\boldsymbol{\Omega}_1)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_1)} \sum_{l=1}^{\rho(\boldsymbol{\Omega}_2)} \sum_{m=1}^{\tau_l(\boldsymbol{\Omega}_2)} \chi_{i,j}(\boldsymbol{\Omega}_1)\chi_{l,m}(\boldsymbol{\Omega}_2)\\ &\times\frac{jm\Gamma(j)\Gamma(m)}{\Gamma(j+m+1)}\left(1 + \frac{\Omega_{1,[i]}\gamma}{\Omega_{1,0}}\right)^{-m}\left(1 + \frac{\Omega_{2,[l]}\gamma}{\Omega_{2,0}}\right)^{-j}\\ &\times\left[\left(\frac{j\Omega_{1,[i]}}{\Omega_{1,0} + \Omega_{1,[i]}\gamma} + \frac{m\Omega_{2,[l]}}{\Omega_{2,0} + \Omega_{2,[l]}\gamma}\right)\right.\\ &\quad\times\, {}_2F_1\left(j, m; j+m+1; \kappa_1(\gamma)\right)\\ &\quad+ \frac{jm\Omega_{1,[i]}\Omega_{2,[l]}\kappa_2(\gamma)\gamma}{(j+m+1)\Omega_{1,0}\Omega_{2,0}}\\ &\quad\left.\times\, {}_2F_1\left(j+1, m+1; j+m+2; \kappa_1(\gamma)\right)\right].\end{aligned} \tag{13}$$

Proof: Taking the derivative of (11) with respect to γ and using the expression for the derivative of the hypergeometric function, which is given in [17, eq. (15.2.1)] as

$$\frac{d}{dz}{}_2F_1(a, b; c; z) = \frac{ab}{c}{}_2F_1(a+1, b+1; c+1; z) \tag{14}$$

we obtain the pdf of γ_2^{AF} in (13). ∎

Using Theorem 2, the outage probability of the dual-hop AF relaying system with multiple interferers can be obtained as

$$\begin{aligned} P_{out}^{AF}(\gamma_{\rm th}) &= \Pr\left[\gamma_2^{AF} < \gamma_{\rm th}\right] = F_{\gamma_2^{AF}}(\gamma_{\rm th}) \\ &= 1 - \sum_{i=1}^{\rho(\boldsymbol{\Omega}_1)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_1)} \sum_{l=1}^{\rho(\boldsymbol{\Omega}_2)} \sum_{m=1}^{\tau_l(\boldsymbol{\Omega}_2)} \chi_{i,j}(\boldsymbol{\Omega}_1)\chi_{l,m}(\boldsymbol{\Omega}_2) \\ &\quad \times \left[\frac{jm\Gamma(j)\Gamma(m)}{\Gamma(j+m+1)}\left(1+\frac{\Omega_{1,[i]}\gamma_{\rm th}}{\Omega_{1,0}}\right)^{-j}\right. \\ &\quad \times \left(1+\frac{\Omega_{2,[l]}\gamma_{\rm th}}{\Omega_{2,0}}\right)^{-m} \\ &\quad \left. \times\, {}_2F_1\left(j,m;j+m+1;\kappa_1(\gamma_{\rm th})\right)\right]. \end{aligned} \tag{15}$$

B. DF Relaying System

For the dual-hop DF relaying system, from (1) and (4), the received SIRs at the relay and the destination are, respectively, given by

$$\gamma_k^{\rm DF} = \frac{|\alpha_k|^2|x_{k,0}|^2}{\sum_{i=1}^{I_k}|\beta_{k,i}|^2|x_{k,i}|^2} = \frac{P_0|\alpha_k|^2}{\sum_{i=1}^{I_k}P_{k,i}|\beta_{k,i}|^2} = X_k, \quad k=1,2. \tag{16}$$

The outage probability of the dual-hop DF relaying system with multiple interferers can be defined as the probability that the minimum of its single-hop SIRs is below a given threshold SIR $\gamma_{\rm th}$. Using the cdf of X_k in Appendix A, the outage probability of the dual-hop DF relaying system with multiple interferers can be obtained as

$$\begin{aligned} P_{out}^{\rm DF}(\gamma_{\rm th}) &= \Pr\left[\min\left(\gamma_1^{\rm DF},\gamma_2^{\rm DF}\right) < \gamma_{\rm th}\right] \\ &= 1 - \Pr\left[\gamma_1^{\rm DF} > \gamma_{\rm th}, \gamma_2^{\rm DF} > \gamma_{\rm th}\right] \\ &= 1 - \left(1 - F_{X_1}(\gamma_{\rm th})\right)\left(1 - F_{X_2}(\gamma_{\rm th})\right) \\ &= 1 - \sum_{i=1}^{\rho(\boldsymbol{\Omega}_1)} \sum_{j=1}^{\tau_i(\boldsymbol{\Omega}_1)} \sum_{l=1}^{\rho(\boldsymbol{\Omega}_2)} \sum_{m=1}^{\tau_l(\boldsymbol{\Omega}_2)} \chi_{i,j}(\boldsymbol{\Omega}_1)\chi_{l,m}(\boldsymbol{\Omega}_2) \\ &\quad \times \left(1+\frac{\Omega_{1,[i]}\gamma_{\rm th}}{\Omega_{1,0}}\right)^{-j}\left(1+\frac{\Omega_{2,[i]}\gamma_{\rm th}}{\Omega_{2,0}}\right)^{-m}. \end{aligned} \tag{17}$$

IV. Numerical Results

Suppose that $\lambda_k = \mu_{k,i} = 1$ for $i = 1,\ldots,I_k$ and that the SIR per hop is defined as $\sigma = P_0/P_I$, where $P_I = \sum_{i=1}^{I_k} P_{k,i}$. Assume co-channel interferers with equal power, that is, $P_{k,i} = P$ for $i = 1,\ldots,I_k$. Fig. 2 shows the outage probability versus SIR per hop for the dual-hop AF and DF relaying systems over Rayleigh fading channels with the threshold SIR $\gamma_{\rm th} = 3$ dB. It is shown that the analytical results perfectly match the simulation results. It is also shown that the outage probability becomes small as the SIR per hop increases or the number of interferers decreases. It is worth noting that the DF relaying system provides better performance than the AF relaying system at low SIR per hop. However, the performance gap between the AF and DF relaying systems becomes small as the SIR per hop increases.

Fig. 3 shows the outage probability versus number of interferers for the dual-hop AF and DF relaying systems over Rayleigh fading channels with $\gamma_{\rm th} = 3$ dB and $\zeta = P_0/P = 25, 30$ dB. It is shown that the analytical results and the simulation results are in excellent agreement. It is also shown that the outage probability becomes large as the number of interferers increases or the SIR per hop decreases. In addition, the outage probability of the DF relaying system is smaller than that of the AF relaying system, and the performance gap between the AF and DF relaying systems becomes large as the number of interferers increases.

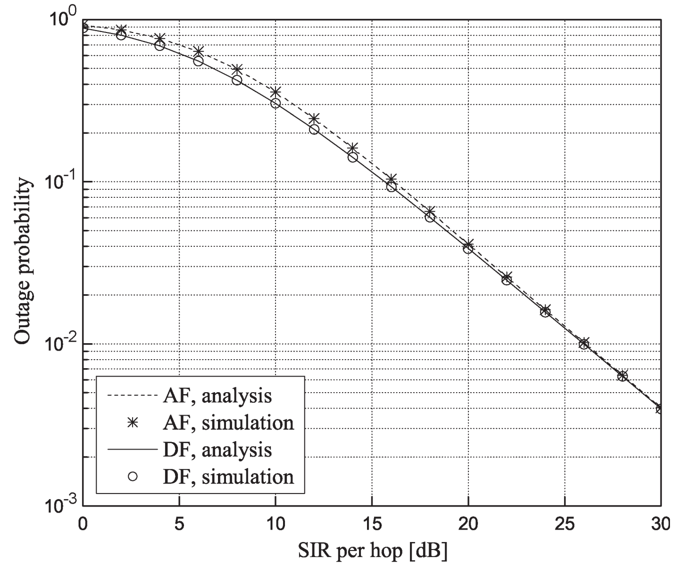

Fig. 2. Outage probability versus SIR per hop over Rayleigh fading channels with $\gamma_{\rm th} = 3$ dB.

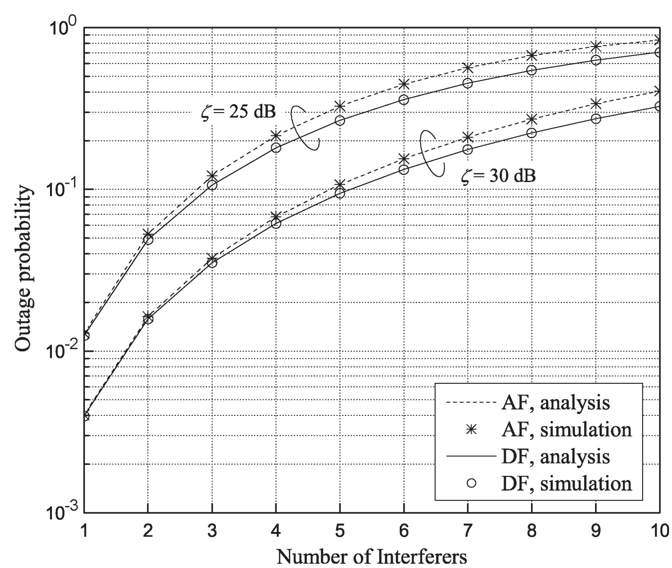

Fig. 3. Outage probability versus number of interferers over Rayleigh fading channels with $\gamma_{\rm th} = 3$ dB and $\zeta = 25, 30$ dB.

V. Conclusion

In this paper, we have investigated the dual-hop AF and DF relaying systems with multiple interferers over Rayleigh fading channels. We derived the closed-form expressions for the outage probability of the dual-hop AF and DF relaying systems with multiple interferers, which have arbitrary transmit power. The validity of our theoretical analysis was verified by comparison with Monte Carlo simulations. In addition, it was shown that the DF relaying system provides better performance than the AF relaying system at low SIR per hop. However, the performance gap between the AF and DF relaying systems becomes small as the SIR per hop increases.

APPENDIX A
PROOF OF PROBABILITY DENSITY FUNCTION OF X_k

The random variable X_k can be rewritten as

$$X_k = \frac{P_0|\alpha_k|^2}{\sum_{i=1}^{I_k} P_{k,i}|\beta_{k,i}|^2} = \frac{A_k}{B_k} \tag{18}$$

where $A_k = P_0|\alpha_k|^2$ is the exponential random variable, and the random variable $B_k = \sum_{i=1}^{I_k} P_{k,i}|\beta_{k,i}|^2$ is the sum of I_k independent but not necessarily identically distributed exponential random variables. Then, the cdf of the exponential random variable A_k is given by

$$F_{A_k}(a) = 1 - \exp\left(-\frac{a}{\Omega_{k,0}}\right). \tag{19}$$

In addition, the pdf of B_k is given by [18]

$$f_{B_k}(b) = \sum_{i=1}^{\rho(\mathbf{\Omega}_k)} \sum_{j=1}^{\tau_i(\mathbf{\Omega}_k)} \chi_{i,j}(\mathbf{\Omega}_k) \frac{\Omega_{k,[i]}^{-j}}{\Gamma(j)} b^{j-1} \exp\left(-\frac{b}{\Omega_{k,[i]}}\right). \tag{20}$$

From (19) and (20), the cdf of X_k is given by

$$\begin{aligned} F_{X_k}(x) &= \Pr\left(\frac{A_k}{B_k} \le x\right) = \mathbb{E}_{\mathrm{B_k}}\left\{F_{X_k|B_k}(x)\right\} \\ &= 1 - \sum_{i=1}^{\rho(\mathbf{\Omega}_k)} \sum_{j=1}^{\tau_i(\mathbf{\Omega}_k)} \chi_{i,j}(\mathbf{\Omega}_k) \frac{\Omega_{k,[i]}^{-j}}{\Gamma(j)} \\ &\quad \times \int_0^\infty b^{j-1} \exp\left[-\left(\frac{x}{\Omega_{k,0}} + \frac{1}{\Omega_{k,[i]}}\right) b\right] db \\ &= 1 - \sum_{i=1}^{\rho(\mathbf{\Omega}_k)} \sum_{j=1}^{\tau_i(\mathbf{\Omega}_k)} \chi_{i,j}(\mathbf{\Omega}_k) \left(1 + \frac{\Omega_{k,[i]}}{\Omega_{k,0}} x\right)^{-j} \end{aligned} \tag{21}$$

where the last equality follows from the fact that $\int_0^\infty e^{-px} x^{q-1} dx = \Gamma(q)/p^q$ for $\Re(p) > 0$ and $\Re(q) > 0$ in [16, eq. (3.385.5)] with $\Re(x)$ denoting the real part of x.

Taking the derivative of (21) with respect to x, we obtain the pdf of X_k in (6). ■

APPENDIX B
PROOF OF CUMULATIVE DISTRIBUTION FUNCTION OF γ_2^{AF}

Let us define a new random variable Z as

$$Z = \frac{1}{X_1} + \frac{1}{X_2}. \tag{22}$$

Under the independence assumption between X_1 and X_2, the MGF of Z is given by $M_Z(s) = M_{Y_1}(s) M_{Y_2}(s)$, where $Y_1 = 1/X_1$, and $Y_2 = 1/X_2$. Using Lemma 2, the MGF of Z is given by

$$\begin{aligned} M_Z(s) = &\sum_{i=1}^{\rho(\mathbf{\Omega}_1)} \sum_{j=1}^{\tau_i(\mathbf{\Omega}_1)} \sum_{l=1}^{\rho(\mathbf{\Omega}_2)} \sum_{m=1}^{\tau_l(\mathbf{\Omega}_2)} \chi_{i,j}(\mathbf{\Omega}_1)\chi_{l,m}(\mathbf{\Omega}_2) \\ &\times jm\Gamma(j)\Gamma(m)\Psi\left(j, 0, \frac{s\Omega_{1,[i]}}{\Omega_{1,0}}\right)\Psi\left(m, 0, \frac{s\Omega_{2,[l]}}{\Omega_{2,0}}\right). \end{aligned} \tag{23}$$

On the other hand, the cdf of γ_2^{AF} can be given by

$$\begin{aligned} F_{\gamma_2^{AF}}(\gamma) &= \Pr\left(\gamma_2^{AF} < \gamma\right) \\ &= \Pr\left(\frac{1}{\gamma_2^{AF}} > \frac{1}{\gamma}\right) = \Pr\left(Z > \frac{1}{\gamma}\right) \\ &= 1 - \Pr\left(Z < \frac{1}{\gamma}\right) = 1 - F_Z\left(\frac{1}{\gamma}\right) \end{aligned} \tag{24}$$

where $F_Z(\cdot)$ is the cdf of Z. Using the differentiation property of Laplace transform, the cdf of Z is given by

$$F_Z(z) = \mathcal{L}^{-1}\left(\frac{M_Z(s)}{s}\right) \tag{25}$$

where $\mathcal{L}^{-1}(\cdot)$ is the inverse Laplace transform.

Substituting (23) into (25) and using (24), we obtain the cdf of γ_2^{AF} as

$$\begin{aligned} &F_{\gamma_2^{AF}}(\gamma) \\ &= 1 - \mathcal{L}^{-1}\Bigg(\sum_{i=1}^{\rho(\mathbf{\Omega}_1)} \sum_{j=1}^{\tau_i(\mathbf{\Omega}_1)} \sum_{l=1}^{\rho(\mathbf{\Omega}_2)} \sum_{m=1}^{\tau_l(\mathbf{\Omega}_2)} \chi_{i,j}(\mathbf{\Omega}_1)\chi_{l,m}(\mathbf{\Omega}_2)\Gamma(j)\Gamma(m) \\ &\qquad \times \left\{\frac{jm}{s}\Psi\left(j, 0, \frac{s\Omega_{1,[i]}}{\Omega_{1,0}}\right)\Psi\left(m, 0, \frac{s\Omega_{2,[l]}}{\Omega_{2,0}}\right)\right\}\Bigg)\Bigg|_{z=1/\gamma}. \end{aligned} \tag{26}$$

With the help of [19, eq. (3.34.6.3)] as

$$\begin{aligned} &\mathcal{L}^{-1}\left(s^{c-1}\Psi(a, c; \sigma s)\Psi(b, c; \omega s)\right) \\ &= \frac{x^{a+b-c}(x+\sigma)^{-a}(x+\omega)^{-b}}{\Gamma(1+a+b-c)} \\ &\quad \times {}_2F_1\left(a, b; 1+a+b-c; \frac{x(x+\sigma+\omega)}{(x+\sigma)(x+\omega)}\right) \end{aligned} \tag{27}$$

we obtain the cdf of γ_2^{AF} in (11). ■

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REFERENCES

[1] H. Yanikomeroglu, “Fixed and mobile relaying technologies for cellular networks,” in *Proc. 2nd Workshop ASWN*, Paris, France, Jul. 2002, pp. 75–81.
[2] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, “Relay-based deployment concepts for wireless and mobile broadband radio,” *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sep. 2004.
[3] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity—Part I: System description,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
[4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[5] M. O. Hasna and M.-S. Alouini, “End-to-end performance of transmission systems with relays over Rayleigh-fading channels,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.
[6] M. O. Hasna and M.-S. Alouini, “Harmonic mean and end-to-end performance of transmission systems with relays,” *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[7] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.
[8] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, Jan. 2006.
[9] D. Lee and J. H. Lee, "Outage probability for opportunistic relaying on multicell environments," in *Proc. IEEE VTC—Spring*, Barcelona, Spain, Apr. 2009, pp. 1–5.
[10] J. Si, Z. Li, and Z. Liu, "Outage probability of opportunistic relaying in Rayleigh fading channels with multiple interferers," *IEEE Signal Process. Lett.*, vol. 17, no. 5, pp. 445–448, May 2010.
[11] C. Zhong, S. Jin, and K. K. Wong, "Dual-hop systems with noisy relay and interference-limited destination," *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 764–768, Mar. 2010.
[12] I. Krikidis, J. Thompson, and S. McLaughlin, "Relay selection issues for amplify-and-forward cooperative systems with interference," in *Proc. IEEE WCNC*, Budapest, Hungary, Apr. 2009, pp. 1–6.
[13] H. Shin and M. Z. Win, "MIMO diversity in the presence of double scattering," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 2976–2996, Jul. 2008.
[14] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 4th ed. New York: McGraw-Hill, 2002.
[15] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed. Hoboken, NJ: Wiley, 2005.
[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. New York: Academic, 2007.
[17] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1972.
[18] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.
[19] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*, vol. 5. New York: Gordon and Breach, 1992.

Queuing Analysis in a Multiuser Diversity System With Adaptive Modulation and Coding Scheme

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***Abstract*—In wireless packet networks, the performance of multiuser diversity schedulers has been analyzed with the saturation assumption; however, the finite queue length effect should be taken into account. We propose a new scheduling scheme exploiting multiuser diversity that considers not only channel information but queue information as well. Considering the finite queue length effect and adaptive modulation and coding scheme (AMCS), we analyze the performance of the proposed scheduler.**

***Index Terms*—Adaptive modulation and coding scheme (AMCS), multiuser diversity, queuing analysis, scheduling.**

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I. Introduction

One of the most notable techniques in increasing network capacity is the multiuser diversity scheduling scheme, which exploits the independence of channel fading among users. For the purpose of efficient spectrum utilization, the multiuser diversity scheduler allows the user with the best channel condition to access the medium on each scheduling instant. However, to fully extract the multiuser diversity gain, the base station (BS) needs to know the channel-quality indicator (CQI) information for every mobile station (MS) in the cell area. Accordingly, feedback information in the network linearly increases as the number of MSs increases. The feedback-reduction scheme is required to mitigate this feedback overhead, and many scheduling schemes [1]–[3] have been proposed with reduced feedback information. In [1], the signal-to-noise ratio (SNR) threshold has been introduced to reduce feedback information. In [2], the quantized CQI has been used to represent an MS's channel state. Moreover, in [3], only one-bit feedback information has been used, and the capacity of this scheduler is close to the proportional fair scheduling scheme [4].

Another issue in schedulers extracting multiuser diversity gain is the fairness problem and performance analysis. The MS located close to the BS actually monopolizes network resources, and the MSs located far from the BS hardly have the chance to access a channel, resulting in a severe fairness problem. As a remedy to this fairness problem, the proportional fair scheduling scheme [4] has been proposed, and it has achieved strict fairness. In [5], the scheduling scheme of adjusting tradeoff between capacity and fairness has been proposed, and this scheme has been extended to the case of selecting multiple MSs [6]. In most performance analysis for the scheduler extracting multiuser diversity gain, all the MSs are assumed to have infinitely backlogged data waiting to be transmitted. As far as the adaptive modulation and coding scheme (AMCS) is concerned, the detailed performance analysis considering finite queue length has been conducted in [7]. This analysis deals with AMCS with a finite queue; however, it does not cover the multiuser diversity gain. In [8], the queuing model for the multiuser diversity scheduler has been analyzed based on the theory of effective bandwidth. However, channel states are simplified into two states.

In this paper, we propose a scheduling scheme that extracts the multiuser diversity gain and uses not only channel-state information but queue information as well. To jointly utilize channel-state information and queue information, the finite queue length effect should be considered in performance analysis. We analyze the performance of the proposed scheduler concerning both AMCS and finite queue length.

II. System Model

Our system is based on the time-division duplex (TDD) mode, and the fixed frame length is assumed. We consider uplink transmission in TDD with total T MSs locating around the BS. The scheduling scheme in this paper can be easily extended to the downlink channel. On each scheduling instant, MSs send CQI information through the feedback channel, and the BS selects a single MS. The selected MS transmits data over the fading channel. In this system, the delay in the signal transmission is not considered, and the error-free feedback channel is assumed. The system model for a scheduler extracting the multiuser diversity gain is depicted in Fig. 1. As aforementioned, AMCS and buffers are adopted in this system. The MSs report CQI information through the feedback channel, and the BS sends AMCS control and selection information to the MSs.