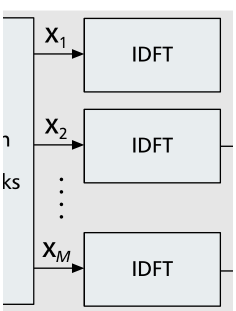
AN OVERVIEW OF PEAK-TO-AVERAGE POWER RATIO REDUCTION TECHNIQUES FOR MULTICARRIER TRANSMISSION

SEUNG HEE HAN, STANFORD UNIVERSITY
JAE HONG LEE, SEOUL NATIONAL UNIVERSITY



High PAPR of the transmit signal is a major drawback of multicarrier transmission such as OFDM or DMT. The authors describe some of the important PAPR reduction techniques for multicarrier transmission.

This work was supported in part by the ITRC Program of the Korean Ministry of Information and Communications and the Brain Korea 21 Project.

ABSTRACT

High peak-to-average power ratio of the transmit signal is a major drawback of multicarrier transmission such as OFDM or DMT. This article describes some of the important PAPR reduction techniques for multicarrier transmission including amplitude clipping and filtering, coding, partial transmit sequence, selected mapping, interleaving, tone reservation, tone injection, and active constellation extension. Also, we make some remarks on the criteria for PAPR reduction technique selection and briefly address the problem of PAPR reduction in OFDMA and MIMO-OFDM.

INTRODUCTION

Multicarrier transmission, also known as orthogonal frequency-division multiplexing (OFDM) or discrete multitone (DMT), is a technique with a long history [1–6] that has recently seen rising popularity in wireless and wireline applications [7–9]. The recent interest in this technique is mainly due to the recent advances in digital signal processing technology. International standards making use of OFDM for high-speed wireless communications are already established or being established by IEEE 802.11, IEEE 802.16, IEEE 802.20, and European Telecommunications Standards Institute (ETSI) Broadcast Radio Access Network (BRAN) committees. For wireless applications, an OFDM-based system can be of interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation can be realized using fast Fourier transform (FFT) techniques.

One of the major drawbacks of multicarrier transmission is the high peak-to-average power ratio (PAPR) of the transmit signal. If the peak transmit power is limited by either regulatory or application constraints, the effect is to reducee the average power allowed under multicarrier transmission relative to that under constant power modulation techniques. This in turn reduces the range of multicarrier transmission. Moreover, to prevent spectral growth of the multicarrier signal in the form of intermodulation among subcarriers and out-of-band radiation, the transmit power amplifier must be operated in its linear region (i.e., with a large input backoff), where the power conversion is inefficient. This may have a deleterious effect on battery lifetime in mobile applications. In many low-cost applications, the drawback of high PAPR may outweigh all the potential benefits of multicarrier transmission systems.

A number of approaches have been proposed to deal with the PAPR problem. These techniques include amplitude clipping [10], clipping and filtering [11, 12], coding [13–21], tone reservation (TR) [22], tone injection (TI) [22], active constellation extension (ACE) [23], and multiple signal representation techniques such as partial transmit sequence (PTS) [24–30], selected mapping (SLM) [30–32], and interleaving [33–35]. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, computational complexity increase, and so on.

In this article we describe some important PAPR reduction techniques for multicarrier transmission with a few illustrative examples. We also mention some of the criteria for selecting a PAPR reduction technique. Finally, we briefly discuss PAPR reduction in orthogonal frequency-division multiple access (OFDMA) and multiple-input multiple-output OFDM (MIMO-OFDM).

In fact, the PAPR problem also arises in many cases other than multicarrier transmission. Typically, the PAPR is not an issue with constant amplitude signals. With nonconstant amplitude signals, however, it is important to deal with the PAPR of those signals. For example, a DS-CDMA signal suffers from the PAPR problem especially in the downlink because it is the sum of the signals for many users. In this article, however, we limit our attention to the PAPR problem in multicarrier transmission only.

 1536-1284/05/$20.00 © 2005 IEEE

THE PAPR OF A MULTICARRIER SIGNAL

A multicarrier signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. Let us denote the collection of all *data symbols* X_n, $n = 0, 1, \ldots, N - 1$, as a vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ that will be termed a *data block*. The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n\Delta f t}, 0 \le t < NT, \qquad (1)$$

where $j = \sqrt{-1}$, Δf is the subcarrier spacing, and NT denotes the useful data block period. In OFDM the subcarriers are chosen to be orthogonal (i.e., $\Delta f = 1/NT$).

The PAPR of the transmit signal is defined as

$$PAPR = \frac{\max\limits_{0 \le t < NT} |x(t)|^2}{1/NT \cdot \int_0^{NT} |x(t)|^2 dt}. \qquad (2)$$

In the remaining part of this article, an approximation will be made in that only NL equidistant samples of $x(t)$ will be considered where L is an integer that is larger than or equal to 1. These "L-times oversampled" time-domain signal samples are represented as a vector $\mathbf{x} = [x0, x1, \ldots, x_{NL-1}]^T$ and obtained as

$$x_k = x(k \cdot T/L) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi kn\Delta f T/L}, \quad k = 0, 1, \ldots NL-1. \qquad (3)$$

It can be seen that the sequence $\{x_k\}$ can be interpreted as the inverse discrete Fourier transform (IDFT) of data block $\mathbf{X}$ with $(L - 1)N$ zero padding. It is well known that the PAPR of the continuous-time signal cannot be obtained precisely by the use of Nyquist rate sampling, which corresponds to the case of $L = 1$. It is shown in [36] that $L = 4$ can provide sufficiently accurate PAPR results. The PAPR computed from the L-times oversampled time domain signal samples is given by

$$PAPR = \frac{\max\limits_{0 \le k \le NL-1} |x_k|^2}{E\left[|x_k|^2\right]}, \qquad (4)$$

where $E[\cdot]$ denotes expectation.

THE CCDF OF THE PAPR

The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. In the literature, the complementary CDF (CCDF) is commonly used instead of the CDF itself. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. In [37] a simple approximate expression is derived for the CCDF of the PAPR of a multicarrier signal with Nyquist rate sampling. From the central limit theorem, the real and imaginary parts of the time domain signal samples follow Gaussian distributions, each with a mean of zero and a variance of 0.5 for a multicarrier signal with a large number of subcarriers. Hence, the amplitude of a multicarrier signal has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom. The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(z). \qquad (5)$$

What we want to derive is the CCDF of the PAPR of a data block. The CCDF of the PAPR of a data block with Nyquist rate sampling is derived as

$$\begin{aligned} P(\mathrm{PAPR} > z) &= 1 - P(\mathrm{PAPR} \le z) \\ &= 1 - F(z)^N \\ &= 1 - (1 - \exp(-z))^N. \end{aligned} \qquad (6)$$

This expression assumes that the N time domain signal samples are mutually independent and uncorrelated. This is not true, however, when oversampling is applied. Also, this expression is not accurate for a small number of subcarriers since a Gaussian assumption does not hold in this case. Therefore, there have been many attempts to derive more accurate distribution of PAPR. Refer to [38–41] for more results on this topic.

The CCDFs are usually compared in a graph such as Fig. 1, which shows the CCDFs of the PAPR of an OFDM signal with 256 and 1024 subcarriers ($N = 256{,}1024$) for quaternary phase shift keying (QPSK) modulation and oversampling factor 4 ($L = 4$). The CCDFs of the PAPR after applying one of the PAPR reduction techniques (i.e., the selected mapping, SLM, technique with 16 candidates) are also shown in Fig. 1. For details of the SLM technique, see the next section. The horizontal and vertical axes represent the threshold for the PAPR and the probability that the PAPR of a data block exceeds the threshold, respectively. It is shown that the unmodified OFDM signal has a PAPR that exceeds 11.3 dB for less than 0.1 percent of the data blocks for $N = 256$. In this case, we say that the 0.1 percent PAPR of the unmodified signal is 11.3 dB. The 0.1 percent PAPR of the unmodified signal is 11.7 dB for $N = 1024$. When SLM

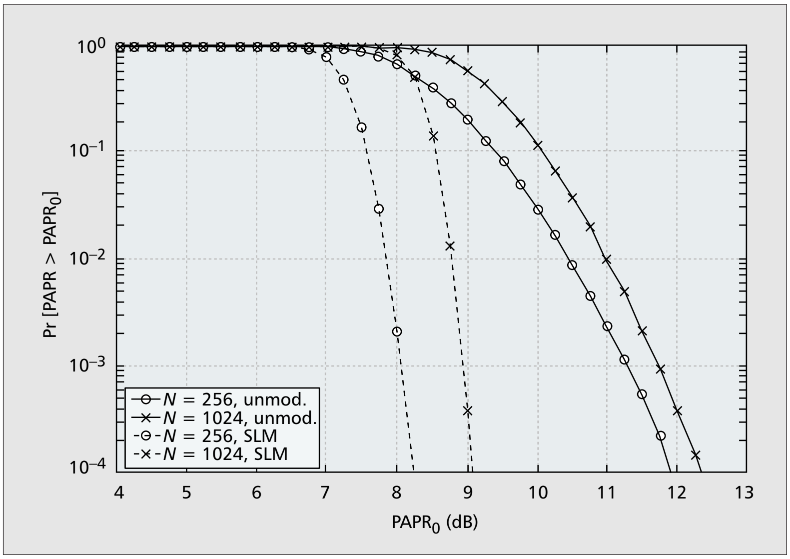


Figure 1. *CCDFs of PAPR of an OFDM signal with 256 and 1024 subcarriers (N = 256,1024) for QPSK modulation and oversampling factor 4 (L = 4).*

Data block X	PAPR (dB)	Data block X	PAPR (dB)
$[1,1,1,1]^T$	6.0	$[1,1,1,1]^T$	2.3
$[1,1,1,-1]^T$	2.3	$[1,1,1,1]^T$	3.7
$[1,1,-1,1]^T$	2.3	$[1,1,1,1]^T$	6.0
$[1,1,-1,-1]^T$	3.7	$[1,1,-1,-1]^T$	2.3
$[1,-1,1,1]^T$	2.3	$[-1,-1,1,1]^T$	3.7
$[1,-1,1,-1]^T$	6.0	$[-1,-1,1,-1]^T$	2.3
$[1,-1,-1,1]^T$	3.7	$[-1,-1,-1,1]^T$	2.3
$[1,-1,-1,-1]^T$	2.3	$[-1,-1,-1,-1]^T$	6.0

Table 1. *PAPR values of all possible data blocks for an OFDM signal with four subcarriers and BPSK modulation.*

is used as a PAPR reduction technique, the 0.1 percent PAPR for $N = 256$ and that for $N = 1024$ reduce to 8.1 dB and 8.9 dB, resulting in 3.2 dB and 2.8 dB reductions, respectively. Speaking roughly, the closer the CCDF curve is to the vertical axis, the better its PAPR characteristic.

PAPR Reduction Techniques for Multicarrier Transmission

In this section we focus more closely on the PAPR reduction techniques for multicarrier transmission with some examples.

Amplitude Clipping and Filtering

The simplest technique for PAPR reduction might be amplitude clipping [10]. Amplitude clipping limits the peak envelope of the input signal to a predetermined value or otherwise passes the input signal through unperturbed [42], that is,

$$B(x)=\begin{cases} x, & |x|\le A \\ Ae^{j\phi(x)}, & |x|\le A \end{cases}, \tag{7}$$

where $\phi(x)$ is the phase of x. The distortion caused by amplitude clipping can be viewed as another source of noise. The noise caused by amplitude clipping falls both in-band and out-of-band. In-band distortion cannot be reduced by filtering and results in an error performance degradation, while out-of-band radiation reduces spectral efficiency. Filtering after clipping can reduce out-of-band radiation but may also cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce overall peak regrowth, a repeated clipping-and-filtering operation can be used [11, 12]. Generally, repeated clipping-and-filtering takes many iterations to reach a desired amplitude level. When repeated clipping-and-filtering is used in conjunction with other PAPR reduction techniques described below, the deleterious effects may be significantly reduced.

There are a few techniques proposed to mitigate the harmful effects of the amplitude clipping. In [43] a method to iteratively reconstruct the signal before clipping is proposed. This method is based on the fact that the effect of clipping noise is mitigated when decisions are made in the frequency domain. When the decisions are converted back to the time domain, the signal is recovered somewhat from the harmful effects of clipping, although this may not be perfect. An improvement can be made by repeating the above procedures. Another way to compensate for the performance degradation from clipping is to reconstruct the clipped samples based on the other samples in the oversampled signals. In [44] oversampled signal reconstruction is used to compensate for signal-to-noise ratio (SNR) degradation due to clipping for low values of clipping threshold. In [45] iterative estimation and cancellation of clipping noise is proposed. This technique exploits the fact that clipping noise is generated by a known process that can be recreated at the receiver and subsequently removed.

Coding

Coding can also be used to reduce the PAPR. A simple idea introduced in [13] is to select those codewords that minimize or reduce the PAPR for transmission. This idea is illustrated in the following example.

Example: The PAPR for all possible data blocks for an OFDM signal with four subcarriers and binary phase shift keying (BPSK) modulation is shown in Table 1. It can be seen from this table that four data blocks result in a PAPR of 6.0 dB, and another four data blocks result in a PAPR of 3.7 dB. It is clear that we could reduce PAPR by avoiding transmitting those sequences. This can be done by block coding the data such that the 3-bit data word is mapped onto a 4-bit codeword such that the set of permissible sequences does not contain those that result in high PAPR. The PAPR of the resulting signal is 2.3 dB, a reduction of 3.7 dB from that without block coding. □

However, this approach suffers from the need to perform an exhaustive search to find the best codes and to store large lookup tables for encoding and decoding, especially for a large number of subcarriers. Moreover, this approach does not address the problem of error correction. A more sophisticated approach proposed in [14] is to use codewords drawn from offsets from a linear code. The idea is to choose the code for its error correcting properties and the offset to reduce the PAPR of the resulting coded signals. This approach enjoys the twin benefits of PAPR reduction and error correction, and is simple to implement, but it requires extensive calculation to find good codes and offsets. A computationally efficient geometrical approach to offset selection is introduced in [15], but there is no guarantee about the amount of PAPR reduction that can be obtained with this approach.

On the other hand, it is discovered that the use of a Golay complementary sequence [16] as codewords to control the modulation results

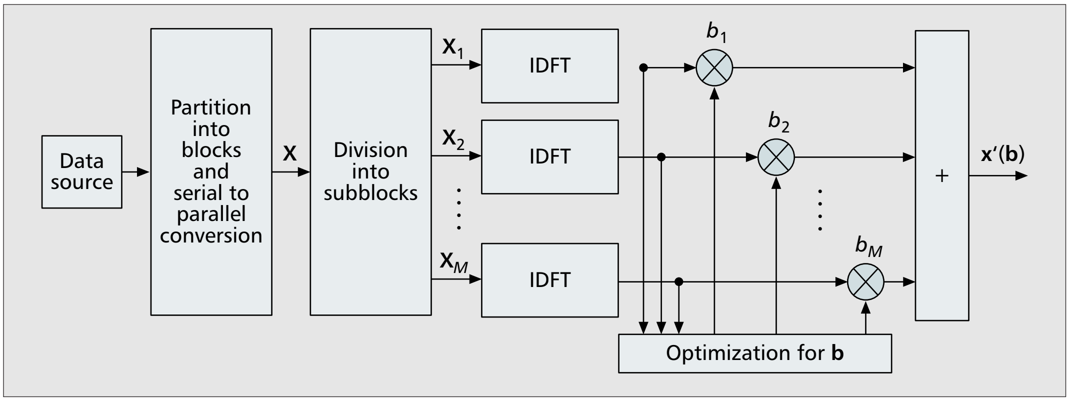


Figure 2. *A block diagram of the PTS technique [24].*

in signals with a PAPR of at most 2. It is found in [17] that the large set of binary length 2^m Golay complementary pairs can be obtained from certain second-order cosets of the classical first-order Reed-Muller code. Thus, it is possible to combine the block coding approach (with all of the encoding, decoding, and error correcting capability) and the use of Golay complementary sequences (with their attractive PAPR control properties). Further improvements and extensions to this approach can be found in [18–21]. However, they can be only applied to MPSK modulation and become infeasible for larger values of N due to the computations needed.

Considering that the usefulness of these techniques is limited to multicarrier systems with a small number of subcarriers and the required exhaustive search for a good code is intractable, the actual benefits of coding for PAPR reduction for practical multicarrier systems are limited.

The Partial Transmit Sequence Technique

In the PTS technique, an input data block of N symbols is partitioned into disjoint subblocks. The subcarriers in each subblock are weighted by a phase factor for that subblock. The phase factors are selected such that the PAPR of the combined signal is minimized. Figure 2 shows the block diagram of the PTS technique. In the ordinary PTS technique [24, 25] input data block $\mathbf{X}$ is partitioned into M disjoint subblocks $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \ldots, X_{m,N-1}]^T$, $m = 1, 2, \ldots, M$, such that $\Sigma_{m=1}^{M}\mathbf{X}_m = \mathbf{X}$ and the subblocks are combined to minimize the PAPR in the time domain. The L-times oversampled time domain signal of $\mathbf{X}_m$, $m = 1, 2, \ldots, M$, is denoted $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \ldots, x_{m,NL-1}]^T$. $\mathbf{x}_m$, $m = 1, 2, \ldots, M$, is obtained by taking an IDFT of length NL on $\mathbf{X}_m$ concatenated with $(L-1)N$ zeros. These are called the partial transmit sequences. Complex phase factors, $b_m = e^{j\phi_m}$, $m = 1, 2, \ldots, M$, are introduced to combine the PTSs. The set of phase factors is denoted as a vector $\mathbf{b} = [b_1, b_2, \ldots, b_M]^T$. The time domain signal after combining is given by

$$\mathbf{x}'(\mathbf{b}) = \sum_{m=1}^{M} b_m \cdot \mathbf{x}_m, \qquad (8)$$

where $\mathbf{x}'(\mathbf{b}) = [x_0'(\mathbf{b}), x_1'(\mathbf{b}). \ldots x_{NL-1}'(\mathbf{b})]^T$. The objective is to find the set of phase factors that minimizes the PAPR. Minimization of PAPR is related to the minimization of

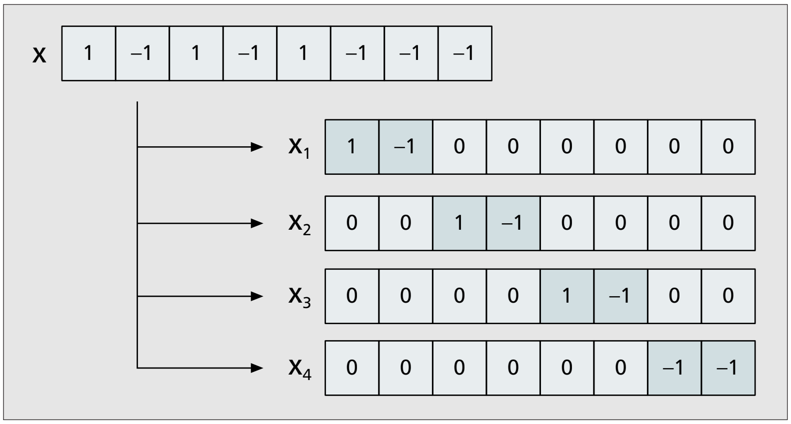
$$\max_{0 \le k \le NL-1} \left| x_k'(\mathbf{b}) \right|.$$

In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is written as $P = \{e^{j2\pi l/W} \mid l = 0, 1, \ldots, W-1\}$, where W is the number of allowed phase factors. In addition, we can set $b_1 = 1$ without any loss of performance. So, we should perform an exhaustive search for $(M-1)$ phase factors. Hence, W^{M-1} sets of phase factors are searched to find the optimum set of phase factors. The search complexity increases exponentially with the number of subblocks M. PTS needs M IDFT operations for each data block, and the number of required side information bits is $\lfloor \log_2 W^{M-1} \rfloor$, where $\lfloor y \rfloor$ denotes the smallest integer that does not exceed y. The amount of PAPR reduction depends on the number of subblocks M and the number of allowed phase factors W. Another factor that may affect the PAPR reduction performance in PTS is the subblock partitioning, which is the method of division of the subcarriers into multiple disjoint subblocks. There are three kinds of subblock partitioning schemes: adjacent, interleaved, and pseudo-random partitioning [25]. Among them, pseudo-random partitioning has been found to be the best choice. The PTS technique works with an arbitrary number of subcarriers and any modulation scheme.

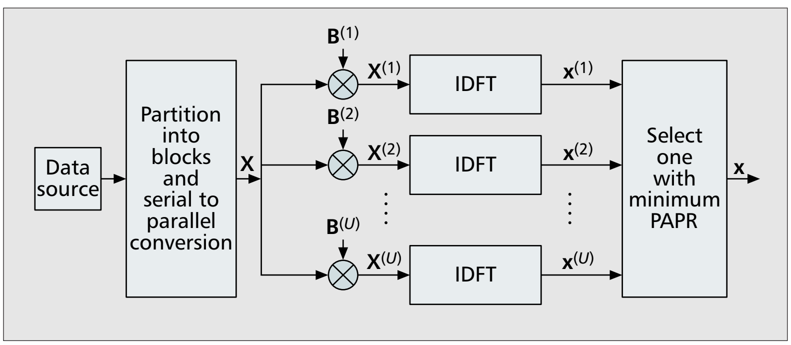
As mentioned above, the ordinary PTS technique has exponentially increasing search complexity. To reduce the search complexity, various techniques have been suggested. In [26] iterations for updating the set of phase factors are stopped once the PAPR drops below a preset threshold. In [27–29] various methods to reduce the number of iterations are presented. These methods achieve significant reduction in search complexity with marginal PAPR performance degradation.

Example: Here, we show a simple example of the PTS technique for an OFDM system with eight subcarriers that are divided into four subblocks. The phase factors are selected in $P = \{\pm 1\}$. Figure 3 shows the adjacent subblock partitioning for a data block $\mathbf{X}$ of length 8. The

Considering that the usefulness of these techniques is limited to multicarrier systems with a small number of subcarriers and the required exhaustive search for a good code is intractable, the actual benefits of coding for PAPR reduction for practical multicarrier systems are limited.



■ **Figure 3**. *An example of adjacent subblock partitioning in PTS.*



■ **Figure 4**. *A block diagram of the SLM technique [30].*

original data block **X** has a PAPR of 6.5 dB. There are 8 (= 2^{4-1}) ways to combine the subblocks with fixed $b_1 = 1$. Among them $[b_1, b_2, b_3, b_4]^T = [1, -1, -1, -1]^T$ achieves the lowest PAPR. The modified data block will be $\mathbf{X}' = \Sigma_{m=1}^{M} b_m \mathbf{X}_m = [1, -1, -1, 1, -1, 1, 1, 1]^T$ whose PAPR is 2.2 dB, resulting in a 4.3 dB reduction. In this case, the number of required IDFT operations is 4 and the amount of side information is 3 bits. The side information must be transmitted to the receiver to recover the original data block. One way to do this is to transmit these side information bits with a separate channel other than the data channel. It is also possible to include the side information within the data block; however, this results in data rate loss. □

The Selected Mapping Technique

In the SLM technique, the transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and selects the most favorable for transmission [30, 31]. A block diagram of the SLM technique is shown in Fig. 4. Each data block is multiplied by U different phase sequences, each of length N, $\mathbf{B}^{(u)} = [b_{u,0}, b_{u,1}, \ldots, b_{u,N-1}]^T$, $u = 1, 2, \ldots, U$, resulting in U modified data blocks. To include the unmodified data block in the set of modified data blocks, we set $\mathbf{B}^{(1)}$ as the all-one vector of length N. Let us denote the modified data block for the uth phase sequence $\mathbf{X}^{(u)} = [X_0 b_{u,0}, X_1 b_{u,1}, \ldots, X_{N-1} b_{u,N-1}]^T$, $u = 1, 2, \ldots, U$. After applying SLM to **X**, the multicarrier signal becomes

$$x^{(u)}(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n b_{u,n} \cdot e^{j2\pi n\Delta ft}, \quad 0 \le t < NT, u = 1, 2, \ldots U. \tag{9}$$

Among the modified data blocks $\mathbf{X}^{(u)}$, $u = 1, 2, \ldots, U$, the one with the lowest PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information. At the receiver, the reverse operation is performed to recover the original data block. For implementation, the SLM technique needs U IDFT operations, and the number of required side information bits is $\lfloor \log_2 U \rfloor$ for each data block. This approach is applicable with all types of modulation and any number of subcarriers. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the design of the phase sequences. In [32] an SLM technique without explicit side information is proposed.

Example: Here, we show a simple example of the SLM technique for an OFDM system with eight subcarriers. We set the number of phase sequences to $U = 4$. The data block to be transmitted is denoted $\mathbf{X} = [1, -1, 1, 1, 1, -1, 1, -1]^T$ whose PAPR before applying SLM is 6.5 dB. We set the four phase factors as $\mathbf{B}^{(1)} = [1, 1, 1, 1, 1, 1, 1, 1]^T$, $\mathbf{B}^{(2)} = [-1, -1, 1, 1, 1, 1, 1, -1]^T$, $\mathbf{B}^{(3)} = [-1, 1, -1, 1, -1, 1, 1, 1]^T$, and $\mathbf{B}^{(4)} = [1, 1, -1, 1, 1, -1, 1, 1]^T$. Among the four modified data blocks $\mathbf{X}^{(u)}$, $u = 1, 2, 3, 4$, $\mathbf{X}^{(2)}$, has the lowest PAPR of 3.0 dB. Hence, $\mathbf{X}^{(2)}$ is selected and transmitted to the receiver. For this data block, the PAPR is reduced from 6.5 to 3.0 dB, resulting in a 3.5 dB reduction. In this case, the number of IDFT operations is 4 and the amount of side information is 2 bits. The amount of PAPR reduction may vary from data block to data block, but PAPR reduction is possible for all data blocks. □

The Interleaving Technique

The interleaving technique for PAPR reduction is very similar to the SLM technique. In this approach, a set of interleavers is used to reduce the PAPR of the multicarrier signal instead of a set of phase sequences [33–35]. An interleaver is a device that operates on a block of N symbols and reorders or permutes them; thus, data block $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ becomes $\mathbf{X}' = [X_{\pi(0)}, X_{\pi(1)}, \ldots, X_{\pi(N-1)}]^T$ where $\{n\} \leftrightarrow \{\pi(n)\}$ is a one-to-one mapping $\pi(n) \in \{0, 1, \ldots, N-1\}$ and for all n. To make K modified data blocks, interleavers are used to produce permuted data blocks from the same data block. The PAPR of $(K-1)$ permuted data blocks and that of the original data block are computed using K IDFT operations; the data block with the lowest PAPR is then chosen for transmission. To recover the original data block, the receiver need only know which interleaver is used at the transmitter; thus, the number of required side information bits is $\lfloor \log_2 K \rfloor$. Both the transmitter and receiver store the permutation indices $\{\pi(n)\}$ in memory. Thus, interleaving and deinterleaving can be done simply. The amount of PAPR reduction depends on the number of interleavers $(K-1)$ and the design of the interleavers.

THE TONE RESERVATION TECHNIQUE

Tone reservation (TR) and tone interjection (TI), explained below, are two efficient techniques to reduce the PAPR of a multicarrier signal. These methods are based on adding a data-block-dependent time domain signal to the original multicarrier signal to reduce its peaks. This time domain signal can be easily computed at the transmitter and stripped off at the receiver.

For the TR technique, the transmitter does not send data on a small subset of subcarriers that are optimized for PAPR reduction [22]. The objective is to find the time domain signal to be added to the original time domain signal **x** such that the PAPR is reduced. If we add a frequency domain vector $\mathbf{C} = [C_0, C_1, \ldots, C_{N-1}]^T$ to $\mathbf{X}$, the new time domain signal can be represented as $\mathbf{x} + \mathbf{c} = \mathrm{IDFT}\{\mathbf{X} + \mathbf{C}\}$, where $\mathbf{c}$ is the time domain signal due to $\mathbf{C}$. The TR technique restricts the data block $\mathbf{X}$ and peak reduction vector $\mathbf{C}$ to lie in disjoint frequency subspaces (i.e., $X_n = 0$, $n \in \{i_1, i_2, \ldots, i_L\}$ and $C_n = 0$, $n \notin \{i_1, i_2, \ldots, i_L\}$). The L nonzero positions in $\mathbf{C}$ are called peak reduction carriers (PRCs). Since the subcarriers are orthogonal, these additional signals cause no distortion on the data bearing subcarriers. To find the value of C_n, $n \in \{i_1, i_2, \ldots, i_L\}$, we must solve a convex optimization problem that can easily be cast as a linear programming (LP) problem. To reduce the computational complexity of LP, a simple gradient algorithm is also proposed in [22].

In the case of DMT for wireline systems, there are typically subcarriers with SNRs too low for sending any information, so these subcarriers must go unused and are available for PAPR reduction. In wireless systems, however, there is typically no fast reliable channel state feedback to dictate whether some subcarriers should not be used. Instead, a set of subcarriers must be reserved regardless of received SNRs, resulting in a bandwidth sacrifice.

THE TONE INJECTION TECHNIQUE

The basic idea here is to increase the constellation size so that each of the points in the original basic constellation can be mapped into several equivalent points in the expanded constellation [22]. Since each symbol in a data block can be mapped into one of several equivalent constellation points, these extra degrees of freedom can be exploited for PAPR reduction. This method is called tone injection because substituting a point in the basic constellation for a new point in the larger constellation is equivalent to injecting a tone of the appropriate frequency and phase in the multicarrier signal.

Assume that M-ary square quadrature amplitude modulation (QAM) is used as a modulation scheme and the minimum distance between constellation points is d. Then the real part of X_n, R_n, and the imaginary part, I_n, can take values $\{\pm d/2, \pm 3d/2, \ldots, \pm(\sqrt{M} - 1)d/2\}$ where $\sqrt{M}$ is equal to the number of levels per dimension. Assume that $X_n = d/2 + j \cdot 3d/2$. Modifying the real and/or imaginary part of X_n could reduce the PAPR of the transmit signal. Since we want the receiver to decode X_n correctly, we must change X_n by an amount that can be estimated at the receiver. A simple case would be to transmit $X_n = X_n + pD + j \cdot qD$, where p and q are any integer values and D is a positive real number known at the receiver. According to [22], the value of D should be at least $d\sqrt{M}$ in order not to increase BER at the receiver. Generally these equivalent signal points are spaced by $D = \rho d \sqrt{M}$ with $\rho \geq 1$. A simple algorithm to find the appropriate subcarrier positions to be modified and the value of p, q is also given in [22]. The only addition to the standard receiver is a modulo-D operation after the symbol decision. The amount of PAPR reduction depends on the value of ρ and the number of modified symbols in a data block.

The TI technique may be more problematic than the TR technique since the injected signal occupies the same frequency band as the information bearing signal. The TI technique may also result in a power increase in the transmit signal due to the injected signal.

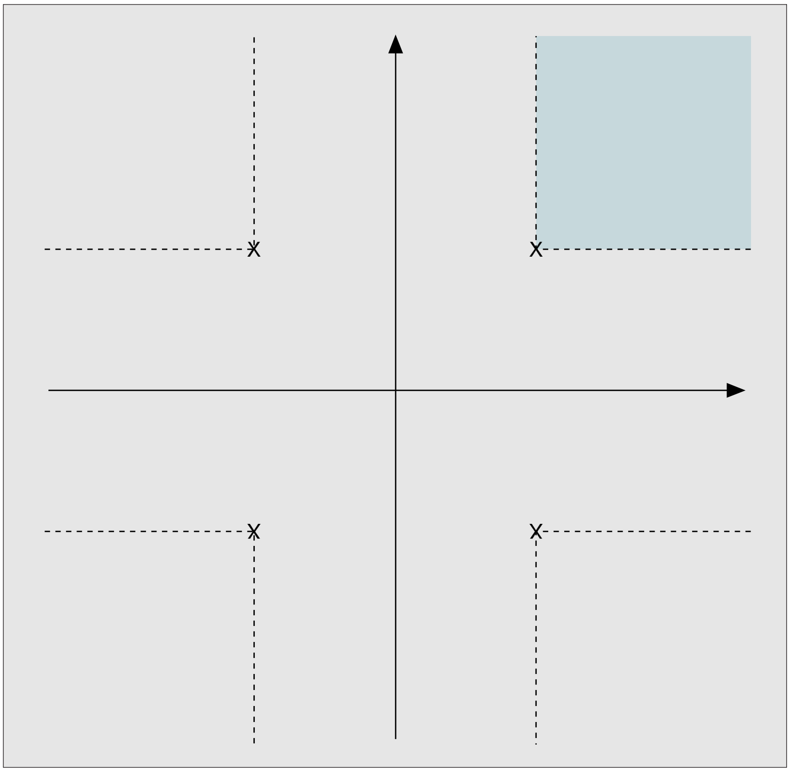


Figure 5. *The ACE technique for QPSK modulation [23].*

THE ACTIVE CONSTELLATION EXTENSION TECHNIQUE

Active constellation extension (ACE) is a PAPR reduction technique similar to TI [23]. In this technique, some of the outer signal constellation points in the data block are dynamically extended toward the outside of the original constellation such that the PAPR of the data block is reduced. The main idea of this scheme is easily explained in the case of a multicarrier signal with QPSK modulation in each subcarrier. In each subcarrier there are four possible constellation points that lie in each quadrant in the complex plane and are equidistant from the real and

There are many factors to consider before a specific PAPR reduction technique is chosen. These factors include PAPR reduction capability, power increase in transmit signal, BER increase at the receiver, loss in data rate, computational complexity increase, and so on.

imaginary axes. Assuming white Gaussian noise, the maximum likelihood decision regions are the four quadrants bounded by the axes; thus, a received data symbol is decided according to the quadrant in which the symbol is observed. Any point that is farther from the decision boundaries than the nominal constellation point (in the proper quadrant) will offer increased margin, which guarantees a lower BER. We can therefore allow modification of constellation points within the quarter-plane outside of the nominal constellation point with no degradation in performance. This principle is illustrated in Fig. 5, where the shaded region represents the region of increased margin for the data symbol in the first quadrant. If adjusted intelligently, a combination of these additional signals can be used to partially cancel time domain peaks in the transmit signal. The ACE idea can be applied to other constellations as well, such as QAM and MPSK constellations, because data points that lie on the outer boundaries of the constellations have room for increased margin without degrading the error probability for other data symbols. This scheme simultaneously decreases the BER slightly while substantially reducing the peak magnitude of a data block. Furthermore, there is no loss in data rate and no side information is required. However, these modifications increase the transmit signal power for the data block, and the usefulness of this scheme is rather restricted for a modulation with a large constellation size.

It is possible to combine the TR and ACE techniques to make the convergence of TR much faster [46].

Other Techniques

It is also possible to avoid high-PAPR signals by employing a technique named *clustered OFDM* [47–49]. In this technique the subcarriers are clustered into several smaller blocks and transmitted over separate antennas. The PAPR is reduced since there are fewer subcarriers per transmitter. However, it has not been widely employed since the increase in the number of power amplifiers makes this proposal impractical in many applications.

Another approach proposed in [50] uses two-dimensional pilot symbol assisted modulation (2D-PSAM), which is usually employed in coherent OFDM for channel estimation, for distortionless PAPR reduction as well as channel estimation. By properly designing the pilot sequence, the complexity of the scheme and the amount of side information can be reduced.

Criteria for Selection of PAPR Reduction Technique

As in everyday life, we must pay some costs for PAPR reduction. There are many factors that should be considered before a specific PAPR reduction technique is chosen. These factors include PAPR reduction capability, power increase in transmit signal, BER increase at the receiver, loss in data rate, computational complexity increase, and so on. Next, we briefly discuss each item.

PAPR reduction capability: Clearly, this is the most important factor in choosing a PAPR reduction technique. Careful attention must be paid to the fact that some techniques result in other harmful effects. For example, the amplitude clipping technique clearly removes the time domain signal peaks, but results in in-band distortion and out-of-band radiation.

Power increase in transmit signal: Some techniques require a power increase in the transmit signal after using PAPR reduction techniques. For example, TR requires more signal power because some of its power must be used for the PRCs. TI uses a set of equivalent constellation points for an original constellation point to reduce PAPR. Since all the equivalent constellation points require more power than the original constellation point, the transmit signal will have more power after applying TI. When the transmit signal power should be equal to or less than that before using a PAPR reduction technique, the transmit signal should be normalized back to the original power level, resulting in BER performance degradation for these techniques.

BER increase at the receiver: This is also an important factor and closely related to the power increase in the transmit signal. Some techniques may have an increase in BER at the receiver if the transmit signal power is fixed or equivalently may require larger transmit signal power to maintain the BER after applying the PAPR reduction technique. For example, the BER after applying ACE will be degraded if the transmit signal power is fixed. In some techniques such as SLM, PTS, and interleaving, the entire data block may be lost if the side information is received in error. This may also increase the BER at the receiver.

Loss in data rate: Some techniques require the data rate to be reduced. As shown in the previous example, the block coding technique requires one out of four information symbols to be dedicated to controlling PAPR. In SLM, PTS, and interleaving, the data rate is reduced due to the side information used to inform the receiver of what has been done in the transmitter. In these techniques the side information may be received in error unless some form of protection such as channel coding is employed. When channel coding is used, the loss in data rate due to side information is increased further.

Computational complexity: Computational complexity is another important consideration in choosing a PAPR reduction technique. Techniques such as PTS find a solution for the PAPR reduced signal by using many iterations. The PAPR reduction capability of the interleaving technique is better for a larger number of interleavers. Generally, more complex techniques have better PAPR reduction capability.

Other considerations: Many of the PAPR reduction techniques do not consider the effect of the components in the transmitter such as the transmit filter, digital-to-analog (D/A) converter, and transmit power amplifier. In practice, PAPR reduction techniques can be used only after careful performance and cost analyses for realistic environments.

In Table 2 we summarize the PAPR reduction techniques considered.

	Distortionless	Power increase	Data rate loss	Requires processing at transmitter (Tx) and receiver (Rx)
Clipping and filtering	No	No	No	Tx: Amplitude clipping, filtering Rx: None
Coding	Yes	No	Yes	Tx: Encoding or table search Rx: Decoding or table search
PTS	Yes	No	Yes	Tx: M IDFTs, W^{M-1} complex vector sums Rx: Side information extraction, inverse PTS
SLM	Yes	No	Yes	Tx: U IDFTs Rx: Side information extraction, inverse SLM
Interleaving	Yes	No	Yes	Tx: K IDFTs, $(K-1)$ interleavings Rx: Side information extraction, inverse interleaving
TR	Yes	Yes	Yes	Tx: IDFTs, find value of PRCs Rx: Ignore non-data-bearing subcarriers
TI	Yes	Yes	No	Tx: IDFTs, search for maximum point in time, tones to be modified, value of p and q Rx: Modulo-D operation
ACE	Yes	Yes	No	Tx: IDFTs, projection onto "shaded area" Rx: None

Table 2. *Comparison of PAPR reduction techniques.*

PAPR Reduction for OFDMA and MIMO-OFDM

Since OFDMA and MIMO-OFDM are based on OFDM, the PAPR problem also arises in both cases. In this section, we briefly discuss the PAPR reduction for OFDMA and MIMO-OFDM.

PAPR Reduction for OFDMA

Recently, OFDMA has received much attention due to its applicability to high speed wireless multiple access communication systems. The evolution of OFDM to OFDMA completely preserves the advantages of OFDM. The drawbacks associated with OFDM, however, are also inherited by OFDMA. Hence, OFDMA also suffers from high PAPR.

Some existing PAPR reduction techniques, which were originally designed for OFDM, process the whole data block as one unit, thus making downlink demodulation of OFDMA systems more difficult since only part of the subcarriers in one OFDMA data block are demodulated by each user's receiver [51]. If downlink PAPR reduction is achieved by schemes designed for OFDM, each user has to process the whole data block and then demodulate the assigned subcarriers to extract their own information. This introduces additional processing for each user's receiver. In the following we describe some modifications of PAPR reduction techniques for an OFDMA downlink. The PAPR problem for an OFDMA uplink is not as serious as that for downlink transmission since each user's transmitter modulates its data to only some of the subcarriers in each data block.

PTS for OFDMA: The PTS technique can easily be modified for OFDMA. Subcarriers from one user are grouped into one or more subblocks, and then PTS is applied to subblocks from all users. One subcarrier per subblock is reserved, and the phase factor for the subblock is embedded into this subcarrier. When applying PTS, the reserved subcarrier does not undergo phase rotation, and this reserved subcarrier is used as a reference for each subblock at the receiver. The phase factor for each subblock is extracted from the reserved subcarrier. Using these phase factors, each user recovers the data in the subblocks for that user.

SLM for OFDMA: The SLM technique can also be modified for OFDMA. Some of the subcarriers are dedicated to transmitting side information for SLM. All users use the information on these dedicated subcarriers to obtain information on which phase sequence is used. Using this knowledge, the data for each user can be restored from the subcarriers of that user only.

TR for OFDMA [51]: In the TR technique for OFDM [22], the symbols in PRCs are optimized for the whole data block in both amplitude and phase. On the other hand, a number of PRCs are assigned to each user only in the TR technique for OFDMA. In order to reduce the computational complexity, the PRCs for each user are optimized for the subcarriers of that user only, making the optimization for the whole OFDMA data block suboptimal.

PAPR Reduction for MIMO-OFDM

Multiple transmit and receive antennas can be used to improve the performance and increase the capacity of wireless communications systems. It is shown that when multiple transmit and receive antennas are used to form a MIMO system, the system capacity can be improved by a factor of the minimum number of transmit and

The TR technique reserves a small number of subcarriers for the purpose of PAPR reduction. In the TR for MIMO-OFDM, the positions of these reserved subcarriers are the same for all transmit antennas and known to both the transmitter and the receiver.

receive antennas compared to a single-input single-output (SISO) system with flat Rayleigh fading or narrowband channels [52, 53]. However, for wideband channels OFDM has to be used with MIMO techniques for intersymbol interference mitigation and capacity improvement. This MIMO-OFDM system is investigated in [54–56]. Since MIMO-OFDM systems are based on OFDM, they also suffer from the problem of inherent PAPR. In the following we describe some modifications of the PAPR reduction techniques for MIMO-OFDM with n_t transmit antennas and n_r receive antennas.

PTS for MIMO-OFDM: The extension of the PTS technique can be used to reduce the PAPR of an MIMO-OFDM signal. In the PTS technique for MIMO-OFDM, input data symbols are converted into n_t parallel streams and the PTS technique for OFDM is applied for each stream or antenna with the sets of phase factors being equal for all transmit antennas. The maximum of the PAPR among all the transmit antennas is reduced. Because the side information is the same for all transmit antennas, the amount of the side information per transmit antenna is reduced. The PTS for MIMO-OFDM, however, may not be as effective as that for OFDM since there are n_t transmit antennas to minimize.

TR for MIMO-OFDM: The TR technique can be extended for MIMO-OFDM very easily. The TR technique reserves a small number of subcarriers for the purpose of PAPR reduction. In TR for MIMO-OFDM, the positions of these reserved subcarriers are the same for all transmit antennas, and known to both the transmitter and the receiver. In each transmit antenna, PAPR reduction is performed independently. The received data symbols in the reserved subcarriers are simply ignored at the receiver.

Conclusions

Multicarrier transmission is a very attractive technique for high-speed transmission over a dispersive communication channel. The PAPR problem is one of the important issues to be addressed in developing multicarrier transmission systems. In this article we describe some PAPR reduction techniques for multicarrier transmission. Many promising techniques to reduce PAPR have been proposed, all of which have the potential to provide substantial reduction in PAPR at the cost of loss in data rate, transmit signal power increase, BER increase, computational complexity increase, and so on. No specific PAPR reduction technique is the best solution for all multicarrier transmission systems. Rather, the PAPR reduction technique should be carefully chosen according to various system requirements. In practice, the effect of the transmit filter, D/A converter, and transmit power amplifier must be taken into consideration to choose an appropriate PAPR reduction technique.

Acknowledgments

The authors would like to thank the reviewers and associate editors for their insightful comments and helpful suggestions that significantly improved the quality of this article. The authors would like to express their gratitude to Prof. Leonard J. Cimini, Jr. of University of Delaware for many constructive comments.

References

[1] R. W. Chang, "Synthesis of Band-Limited Orthogonal Signals for Multichannel Data Transmission," *Bell Sys. Tech. J.*, vol. 46, no. 12, Dec. 1966, pp. 1775–96.

[2] B. R. Saltzberg, "Performance of an Efficient Parallel Data Transmission System," *IEEE Trans. Commun.*, vol. 15, no. 6, Dec. 1967, pp. 805–11.

[3] R. W. Chang and R. A. Gibby, "A Theoretical Study of Performance of an Orthogonal Multiplexing Data Transmission Scheme," *IEEE Trans. Commun.*, vol. 16, no. 4, Aug. 1968, pp. 529–40.

[4] S. B. Weinstein and P. M. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform," *IEEE Trans. Commun.*, vol. 19, no. 5, Oct. 1971, pp. 628–34.

[5] L. J. Cimini, Jr., "Analysis and Simulation of a Digital Mobile Channel using Orthogonal Frequency Division Multplexing," *IEEE Trans. Commun.*, vol. 33, no. 7, July 1985, pp. 665–75.

[6] J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come," *IEEE Commun. Mag.*, vol. 28, no. 5, May 1990, pp. 5–14.

[7] M. Alard and R. Lasalle, "Principles of Modulation and Channel Coding for Digital Broadcasting for Mobile Receivers," *EBU Rev.*, vol. 224, Aug. 1987, pp. 47–69.

[8] U. Reimers, "Digital Video Broadcasting," *IEEE Commun. Mag.*, vol. 36, no. 10, June 1998, pp. 104–10.

[9] B. R. Saltzberg, "Comparison of Single-Carrier and Multitone Digital Modulation for ADSL Applications," *IEEE Commun. Mag.*, vol. 36, no. 11, Nov. 1998, pp. 114–21.

[10] R. O'Neill and L. B. Lopes, "Envelope Variations and Spectral Splatter in Clipped Multicarrier Signals," *Proc. IEEE PIMRC '95*, Toronto, Canada, Sept. 1995, pp. 71–75.

[11] X. Li and L. J. Cimini, Jr., "Effect of Clipping and Filtering on the Performance of OFDM," *IEEE Commun. Lett.*, vol. 2, no. 5, May 1998, pp. 131–33.

[12] J. Armstrong, "Peak-to-Average Power Reduction for OFDM by Repeated Clipping and Frequency Domain Filtering," *Elect. Lett.*, vol. 38, no. 8, Feb. 2002, pp. 246–47.

[13] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block Coding Scheme for Reduction of Peak to Mean Envelope Power Ratio of Multicarrier Transmission Scheme," *Elect. Lett.*, vol. 30, no. 22, Dec. 1994, pp. 2098–99.

[14] A. E. Jones and T. A. Wilkinson, "Combined Coding for Error Control and Increased Robustness to System Nonlinearities in OFDM," *Proc. IEEE VTC '96*, Atlanta, GA, Apr. –May 1996, pp. 904–08.

[15] V. Tarokh and H. Jafarkhani, "On the Computation and Reduction of the Peak-to-Average Power Ratio in Multicarrier Communications," *IEEE Trans. Commun.*, vol. 48, no. 1, Jan. 2000, pp. 37–44.

[16] M. Golay, "Complementary Series," *IEEE Trans. Info. Theory*, vol. 7, no. 2, Apr. 1961, pp. 82–87.

[17] J. A. Davis and J. Jedwab, "Peak-to-Mean Power Control and Error Correction for OFDM Transmission Using Golay Sequences and Reed-Muller Codes," *Elect. Lett.*, vol. 33, no. 4, Feb. 1997, pp. 267–68.

[18] J. A. Davis and J. Jedwab, "Peak-to-Mean Power Control in OFDM, Golay Complementary Sequences, and Reed–Muller Codes," *IEEE Trans. Info. Theory*, vol. 45, no. 7, Nov. 1999, pp. 2397–17.

[19] K. Patterson, "Generalized Reed-Muller Codes and Power Control in OFDM Modulation," *IEEE Trans. Info. Theory*, vol. 46, no. 1, Jan. 2000, pp. 104–20.

[20] K. G. Paterson and V. Tarokh, "On the Existence and Construction of Good Codes with Low Peak-to-Average Power Ratios," *IEEE Trans. Info. Theory*, vol. 46, no. 6, Sept. 2000, pp. 1974–87.

[21] C. V. Chong and V. Tarokh, "A Simple Encodable/Decodable OFDM QPSK Code with Low Peak-to-Mean Envelope Power Ration,"*IEEE Trans. Info. Theory*, vol. 47, no. 7, Nov. 2001, pp. 3025–29.

[22] J. Tellado, *Peak to Average Power Reduction for Multicarrier Modulation*, Ph.D. dissertation, Stanford Univ., 2000.

[23] B. S. Krongold and D. L. Jones, "PAR Reduction in OFDM via Active Constellation Extension," *IEEE Trans. Broadcast.*, vol. 49, no. 3, Sept. 2003, pp. 258–68.

[24] S. H. Müller and J. B. Huber, "OFDM with Reduced Peak–to–Average Power Ratio by Optimum Combination of Partial Transmit Sequences," *Elect. Lett.*, vol. 33, no. 5, Feb. 1997, pp. 368–69.

[25] S. H. Müller and J. B. Huber, "A Novel Peak Power Reduction Scheme for OFDM," *Proc. IEEE PIMRC '97*, Helsinki, Finland, Sept. 1997, pp. 1090–94.
[26] A. D. S. Jayalath and C. Tellambura, "Adaptive PTS Approach for Reduction of Peak-to-Average Power Ratio of OFDM Signal," *Elect. Lett.*, vol. 36, no. 14, July 2000, pp. 1226–28.
[27] L. J. Cimini, Jr. and N. R. Sollenberger, "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences," *IEEE Commun. Lett.*, vol. 4, no. 3, Mar. 2000, pp. 86–88.
[28] C. Tellambura, "Improved Phase Factor Computation for the PAR Reduction of an OFDM Signal Using PTS," *IEEE Commun. Lett.*, vol. 5, no. 4, Apr. 2001, pp. 135–37.
[29] S. H. Han and J. H. Lee, "PAPR Reduction of OFDM Signals Using a Reduced Complexity PTS Technique," *IEEE Sig. Proc. Lett.*, vol. 11, no. 11, Nov. 2004, pp. 887–90.
[30] S. H. Müller and J. B. Huber, "A Comparison of Peak Power Reduction Schemes for OFDM," *Proc. IEEE GLOBECOM '97*, Phoenix, AZ, Nov. 1997, pp. 1–5.
[31] R. W. Bäuml, R. F. H. Fisher, and J. B. Huber, "Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selected Mapping," *Elect. Lett.*, vol. 32, no. 22, Oct. 1996, pp. 2056–57.
[32] H. Breiling, S. H. Müller–Weinfurtner, and J. B. Huber, "SLM Peak-Power Reduction without Explicit Side Information," *IEEE Commun. Lett.*, vol. 5, no. 6, June 2001, pp. 239–41.
[33] G. R. Hill, M. Faulkner, and J. Singh, "Reducing the Peak-to-Average Power Ratio in OFDM by Cyclically Shifting Partial Transmit Sequences," *Elect. Lett.*, vol. 36, no. 6, Mar. 2000, pp. 560–61.
[34] P. Van Eetvelt, G. Wade, and M. Tomlinson, "Peak to Average Power Reduction for OFDM Schemes by Selective Scrambling," *Elect. Lett.*, vol. 32, no. 21, Oct. 1996, pp. 1963–64.
[35] A. D. S. Jayalath and C. Tellambura, "Reducing the Peak-to-Average Power Ratio of Orthogonal Frequency Division Multiplexing Signal through Bit or Symbol Interleaving," *Elect. Lett.*, vol. 36, no. 13, June 2000, pp. 1161–63.
[36] C. Tellambura, "Computation of the Continuous–Time PAR of an OFDM Signal with BPSK Subcarriers," *IEEE Commun. Lett.*, vol . 5, no. 5, May 2001, pp. 185–87.
[37] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, 2000.
[38] H. Ochiai and H. Imai, "On the Distribution of the Peak–to–Average Power Ratio in OFDM Signals," *IEEE Trans. Commun.*, vol. 49, no. 2, Feb. 2001, pp. 282–89.
[39] S. Wei, D. L. Goeckel, and P. E. Kelly, "A Modern Extreme Value Theory Approach to Calculating the Distribution of the PAPR in OFDM Systems," *Proc. IEEE ICC 2002*, New York, NY, May 2002, pp. 1686–90.
[40] M. Sharif, M. Gharavi-Alkhansari, and B. H. Khalaj, "On the Peak-to-Average Power of OFDM Signals Based on Oversampling," *IEEE Trans. Commun.*, vol. 51, no. 1, Jan. 2003, pp. 72–78.
[41] G. Wunder and H. Boche, "Upper Bounds on the Statistical Distribution of the Crest-Factor in OFDM Transmission," *IEEE Trans. Info. Theory*, vol. 49, no. 2, Feb. 2003, pp. 488–94.
[42] J. Heiskala and J. Terry, *OFDM Wireless LANs: A Theoretical and Practical Guide*, Sams Publishing, 2002.
[43] D. Kim and G. L. Stüber, "Clipping Noise Mitigation for OFDM by Decision–Aided Reconstruction," *IEEE Commun. Lett.*, vol. 3, no. 1, Jan. 1999, pp. 4–6.
[44] H. Saeedi, M. Sharif, and F. Marvasti, "Clipping Noise Cancellation in OFDM Systems Using Oversampled Signal Reconstruction," *IEEE Commun. Lett.*, vol. 6, no. 2, Feb. 2002, pp. 73–75.
[45] H. Chen and M. Haimovish, "Iterative Estimation and Cancellation of Clipping Noise for OFDM Signals," *IEEE Commun. Lett.*, vol. 7, no. 7, July 2003, pp. 305–07.
[46] B. S. Krongold and D. L. Jones, "An Active Set Approach for OFDM PAR Reduction via Tone Reservation," *IEEE Trans. Sig. Proc.*, vol. 52, no. 2, Feb. 2004, pp. 495–509.
[47] L. J. Cimini, Jr., B. Daneshrad, and N. R. Sollenberger, "Clustered OFDM with Transmitter Diversity and Coding," *Proc. IEEE GLOBECOM '96*, London, U.K., Nov. 1996, pp. 703–07.
[48] L. J. Cimini, Jr., and N. R. Sollenberger, "OFDM with Diversity and Coding for Advanced Cellular Internet Services," *Proc. IEEE GLOBECOM '97*, Phoenix, AZ, Nov. 1997, pp. 305–09.
[49] R. Dinis, P. Montezuma, and A. Gusmao, "Performance Trade-offs with Quasi-Linearly Amplified OFDM through a Two–Branch Combining Technique," *Proc. IEEE VTC '96,* Atlanta, GA, Apr.–May 1996, pp. 899–903.
[50] M. J. Fernández–Getino García, J. M. Páez–Borrallo, and O. Edfors, "Orthogonal Pilot Sequences for Peak-to-Average Power Reduction in OFDM," *Proc. IEEE VTC 2001-Fall*, Atlantic City, NJ, Oct. 2001, pp. 650–54.
[51] Z. Yunjun, A. Yongacoglu, and J.-Y. Chouinard, "Orthogonal Frequency Division Multiple Access Peak–to–Average Power Ratio Reduction Using Optimized Pilot Symbols," *Proc. ICCT 2000*, Beijing, China, Aug. 2000, pp. 574–77.
[52] G. J. Foschini, "Layered Space–Time Architecture for Wireless Communication in a Fading Environment When Using Multi-Element Antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, Autumn 1996, pp. 41–59.
[53] G. J. Foschini and M. J. Gans, "On Limits of Wireless Communications in Fading when Using Multiple Antennas," *Wireless Pers. Commun.*, vol. 6, no. 3, Mar. 1998, pp. 311–35.
[54] Y. Li, J. H. Winters, and N. R. Sollenberger, "MIMO–OFDM for Wireless Communications: Signal Detection with Enhanced Channel Estimation," *IEEE Trans. Commun.*, vol. 50, no. 9, Sept. 2002, pp. 1471–77.
[55] A. J. Paulraj *et al.*, "An Overview of MIMO Communications — a Key to Gigabit Wireless," *Proc. IEEE*, vol. 92, no. 2, Feb. 2004, pp. 198–218.
[56] G. L. Stüber *et al.*, "Broadband MIMO-OFDM Wireless Communications," *Proc. IEEE*, vol. 92, no. 2, Feb. 2004, pp. 271–94.

BIOGRAPHIES

SEUNG HEE HAN [S'98] (shhan75@snu.ac.kr) received B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Korea, in 1998, 2000, and 2005,respectively. Currently, he is with Stanford University as a postdoctoral visiting scholar. His current research interests include multicarrier transmission, spread-spectrum communication, and their application to wireless communications.

JAE HONG LEE [M'86, SM'03] received B.S. and M.S. degrees in electronics engineering from Seoul National University, Korea, in 1976 and 1978, respectively, and a Ph.D. degree in electrical engineering from the University of Michigan, Ann Arbor, in 1986. From 1978 to 1981 he was with the Department of Electronics Engineering, Republic of Korea Naval Academy, Jinhae, as an instructor. In 1987 he joined the faculty of SNU. He was a member of technical staff at AT&T Bell Laboratories, Whippany, New Jersey, from 1991 to 1992. Currently, he is with Seoul National University as a professor in the School of Electrical Engineering. His current research interests include communication and coding theory, code-division multiple access (CDMA), orthogonal frequency-division multiplexing (OFDM), and their application to wireless and satellite communications. He is a member of the Institute of Electronics Engineers of Korea (IEEK), KICS, the Korea Society of Broadcasting Engineers (KSBE), and Tau Beta Pi. He is a Vice President of the IEEK and KSBE.

No specific PAPR reduction technique is the best solution for all multicarrier transmission systems. Rather, the PAPR reduction technique should be carefully chosen according to various system requirements.