PAPR Reduction of OFDM Signals Using a Reduced Complexity PTS Technique

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***Abstract*—A major drawback of orthogonal frequency division multiplexing (OFDM) is the high peak-to-average power ratio (PAPR) of the transmitted signal. Partial transmit sequence (PTS) technique can improve the PAPR statistics of OFDM signals. In the PTS technique, the data block to be transmitted is partitioned into disjoint subblocks and the subblocks are combined using phase factors to minimize PAPR. As ordinary PTS technique requires an exhaustive search over all combinations of allowed phase factors, the search complexity increases exponentially with the number of subblocks. In the proposed technique, a gradient descent search is performed to find the phase factors. It is shown that the proposed technique achieves significant reduction in search complexity with little performance degradation.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is a promising solution for high data rate transmission in frequency-selective fading channels [1]. A major drawback of OFDM at the transmitter side is the high peak-to-average power ratio (PAPR) of the transmitted signal. High peaks of OFDM signals occur when the sinusoidal signals of the subcarriers are added constructively. These high peaks necessitate using larger and expensive linear power amplifiers. Since high peaks occur irregularly and infrequently, this means that power amplifiers will be operating inefficiently.

A large number of solutions have been proposed to solve the PAPR problem in OFDM. Clipping OFDM signal before amplification is a simple solution [2]. However, clipping may cause inter-modulation among subcarriers and undesired out-of-band radiation. Another solution uses block coding [3], where the data sequence is embedded in a larger sequence and only a subset of all the possible sequences are used, specifically, those with low peak powers [4]. While block coding reduces PAPR, it also reduces transmission rate, significantly so for a large number of subcarriers [5]. Furthermore, there is no efficient coding scheme for a large number of subcarriers.

Recently, a promising technique for improving the statistics of the PAPR of OFDM signals has been proposed: the partial transmit sequence (PTS) technique [6]. In the PTS technique, the input data block is broken up into disjoint subblocks. The subblocks are multiplied by phase factors and then added together to produce alternative transmit signals containing the same information. The phase factors, whose amplitude is usually set to 1, are selected such that the resulting PAPR is minimized. The number of allowed phase factors should not be excessively high, in order to keep the number of required side information bits and the search complexity within a reasonable limit. However, the exhaustive search complexity of the ordinary PTS technique increases exponentially with the number of subblocks, so it is practically not realizable for a large number of sub-blocks.

In an effort to simplify the ordinary PTS technique, a recent work [4] has introduced a suboptimal iterative flipping algorithm to find the phase factors. Although the iterative flipping algorithm significantly reduces the search complexity, there is some gap between its PAPR statistic and that of the ordinary PTS technique.

In this letter, we propose a novel PTS technique with reduced complexity. It provides better PAPR statistic than the iterative flipping algorithm with reasonable search complexity. We also show that the proposed technique includes the ordinary PTS technique as a special case.

II. Peak-to-Average Power Ratio

Let us denote the data block of length N as a vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ where N is equal to the number of subcarriers. The duration of a symbol X_n in $\mathbf{X}$ is T. Each symbol in $\mathbf{X}$ modulates one of a set of subcarriers, $\{f_n, n = 0, 1, \ldots, N-1\}$. The N subcarriers are chosen to be orthogonal, that is, $f_n = n\Delta f$, where $\Delta f = 1/NT$ and NT is the duration of the OFDM data block $\mathbf{X}$. The complex envelope of the transmitted OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \quad 0 \le t < NT. \tag{1}$$

The PAPR of the transmitted signal in (1) is defined as

$$\text{PAPR} = \frac{\max_{0 \le t < NT} |x(t)|^2}{1/NT \cdot \int_0^{NT} |x(t)|^2 \, dt}. \tag{2}$$

In principle, PAPR reduction techniques are concerned with reducing $\max|x(t)|$. However, since most systems employ discrete-time signals, the amplitude of samples of $x(t)$ is dealt with in many of the PAPR reduction techniques. Since symbol-spaced sampling of (1) sometimes misses some of the signal peaks and results in optimistic results for the PAPR, signal samples are obtained by oversampling (1) by a factor of L to approx-

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imate the true PAPR better. The L-times oversampled time-domain samples are obtained by an LN-point inverse discrete Fourier transform (IDFT) of the data block with $(L-1)N$ zero-padding. It was shown in [7] that $L = 4$ is sufficient to capture the peaks.

III. PTS Techniques

Fig. 1 shows the block diagram of the PTS techniques. All of three techniques described below can be implemented by appropriately changing the "Optimization for $\mathbf{b}$" block in Fig. 1.

A. Ordinary PTS Technique

In the ordinary PTS technique [6], the input data block $\mathbf{X}$ is partitioned into M disjoint subblocks $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \ldots, X_{m,N-1}]^T, m = 1, 2, \ldots, M$, such that $\sum_{m=1}^{M} \mathbf{X}_m = \mathbf{X}$ and the subblocks are combined to minimize the PAPR in the time-domain. The L-times oversampled time-domain signal of $\mathbf{X}_m, m = 1, 2, \ldots, M$, is denoted as $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \ldots, x_{m,NL-1}]^T$. $\mathbf{x}_m, m = 1, 2, \ldots, M$, is obtained by taking an IDFT of length NL on $\mathbf{X}_m$ concatenated with $(L-1)N$ zeros. These are called as the partial transmit sequences. Complex phase factors, $b_m = e^{j\phi_m}, m = 1, 2, \ldots, M$, are introduced to combine the partial transmit sequences. We shall write the set of the phase factors as a vector $\mathbf{b} = [b_1, b_2, \ldots, b_M]^T$. The time-domain signal after combining is given by

$$\mathbf{x}'(\mathbf{b}) = \sum_{m=1}^{M} b_m \cdot \mathbf{x}_m \tag{3}$$

where $\mathbf{x}'(\mathbf{b}) = [x_0'(\mathbf{b}), x_1'(\mathbf{b}), \ldots, x_{NL-1}'(\mathbf{b})]^T$. The objective is to find the phase factors with the aim of minimizing PAPR. Minimization of PAPR is related to the minimization of $\max_{0 \le k \le NL-1} |x_k'(\mathbf{b})|$. In general, the selection of the phase factors is limited to a set with finite number of elements to reduce the search complexity. The set of allowed phase factors is written as

$$P = \{e^{j2\pi l/W} \mid l = 0, 1, \ldots, W-1\} \tag{4}$$

where W is the number of allowed phase factors. Since we are dealing with the amplitude of the signal samples, one phase factor can be fixed without any performance loss, e.g., $b_1 = 1$. So, we should perform exhaustive search for $(M-1)$ phase factors. Hence, W^{M-1} sets of phase factors are searched to find the optimum set of phase factors. The search complexity increase exponentially with the number of subblocks M.

B. Iterative Flipping Algorithm

In the iterative flipping algorithm [4], each input data block is divided into M subblocks to form partial transmit sequences as in the ordinary PTS technique. Assume that $b_m = 1$ for all m and compute PAPR of the combined signal. As a first step, we fix $b_1 = 1$. Then change the next phase factor b_2 among all possible values in P and recompute the resulting PAPR. Then, choose the value which achieves the lowest PAPR as part of

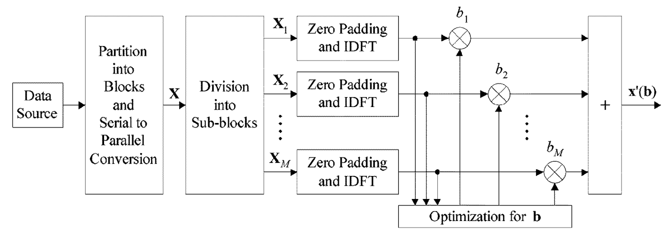

Fig. 1. Block diagram of the PTS techniques.

the final set of phase factors. The algorithm continues in this fashion until all other phase factors have been explored. The name "flipping" came from the fact that the flipping the signs of the phase factors occurs in the case of $W = 2$. The search complexity of the technique is proportional to $(M-1) \cdot W$.

C. Reduced Complexity PTS Technique

Although the iterative flipping algorithm significantly reduces the search complexity, there is some performance gap between the ordinary PTS technique and the iterative flipping algorithm. The iterative flipping algorithm changes the phase factors *one by one* and *simply in the order of increasing* m. If we find better order for changing the phase factors, we might expect improved performance. In addition, if we increase the number of phase factors explored simultaneously, further improvement can be expected.

Here, we propose a novel PTS technique based on gradient descent search which is useful in solving combinatorial optimization problems [8]. The objective of the technique is to find the phase factors that achieve PAPR statistic close to that of the ordinary PTS technique with reduced search complexity and little performance degradation. The proposed technique starts with a pre-determined vector of phase factors. Next, it finds an updated vector of phase factors in its "neighborhood" that results in the largest reduction in PAPR. Neighborhood of radius r is defined as the set of vectors with Hamming distance equal to or less than r from its origin. The equation that updates the vector of phase factors from $\mathbf{b}$ to $\mathbf{b}'$ is given by

$$\mathbf{b}' = \arg\left\{\max_{\|\hat{\mathbf{b}}-\mathbf{b}\|_H \le r} (\text{PAPR for } \mathbf{b} - \text{PAPR for } \hat{\mathbf{b}})\right\} \tag{5}$$

where $\|\cdot\|_H$ denotes the Hamming weight of its vector argument and r denotes the radius of the neighborhood which is centered at $\mathbf{b}$. This process is repeated using the updated vector of phase factors as a new starting point as long as PAPR reduction is achieved. We may limit the number of maximum iterations to update the phase factors. The performance and complexity of the technique is dependent on the value of r. If r is equal to M, the proposed technique searches for all combinations of allowed phase factors. In this case, the proposed technique is equivalent to the ordinary PTS technique. Hence, the proposed technique includes the ordinary PTS technique as a special case. If $r = 1$, it offers search complexity which grows linearly with the number of subblocks M. Various r between 1 and M correspond to algorithms which provide tradeoffs between performance and complexity. The reduced complexity PTS technique is summarized as follows.

1) Partition data block into subblocks.
2) Set $\mathbf{b} = [1, 1, \ldots, 1]^T$[1] and iteration count $i = 1$.
3) Among the vectors of phase factors in the neighborhood of $\mathbf{b}$ (with radius r), find $\mathbf{b}'$ that achieves the smallest PAPR.
4) If PAPR for $\mathbf{b}'$ is smaller than PAPR for $\mathbf{b}$, update $\mathbf{b}$ with $\mathbf{b}'$ and proceed to step 5); otherwise, terminate.
5) If i is smaller than its maximum I, increase i by 1 and go to step 3); otherwise, terminate.

The search complexity of the technique is proportional to ${}_{M-1}C_r \cdot W^r$ where the binomial coefficient ${}_uC_v$ is defined as

$$ {}_uC_v = \binom{u}{v} \triangleq \frac{u!}{(u-v)!v!}. \tag{6} $$

The search complexity of the technique also increases as the number of maximum iterations I increases. When $r = 1$, the technique has search complexity comparable to that of the iterative flipping algorithm. We may lower the search complexity of the proposed technique by employing a PAPR threshold and applying the proposed technique only for data blocks with PAPR larger than the threshold.

IV. Results and Discussions

We assume an OFDM system with 64 subcarriers $(N = 64)$ or 128 subcarriers $(N = 128)$ with QPSK data symbols. Also assume that the number of allowed phase factors is 4 $(W = 4)$ with $P = \{\pm 1, \pm j\}$. We divide the 64 subcarriers into 8 subblocks $(M = 8)$ with eight contiguous subcarriers and the 128 subcarriers into 8 subblocks $(M = 8)$ with 16 contiguous subcarriers. The transmitted signal is oversampled by a factor of 4 $(L = 4)$. 100 000 random OFDM blocks were generated to obtain the complementary cumulative density functions (CCDFs) of PAPR. In the following figures, we will denote the reduced complexity PTS technique by its parameters r and I. For example, "$r2/I3$" denotes the proposed technique with $r = 2$ and $I = 3$.

Fig. 2 shows the CCDFs of PAPR of the reduced complexity PTS technique as a function of the maximum iteration I and the radius of the neighborhood r for $N = 64$. It is shown in Fig. 2 that the reduced complexity PTS technique has a PAPR which exceeds 7.70 dB for less than 0.1% of the blocks for $I = 1$ and $r = 1$. For $I = 2$ and $r = 1$, we can lower the 0.1% PAPR by 0.60 dB than the case with $I = 1$ and $r = 1$. We can lower the 0.1% PAPR by 0.10 dB with additional increase in I. It is shown that there is little improvement in resulting PAPR for further increase in I when $r = 1$. It is shown in Fig. 2 that, when $r = 2$, the 0.1% PAPRs for $I = 1, I = 2$, and $I = 3$ are 6.70, 6.40, and 6.30 dB, respectively. Increasing I beyond 3 seems to bring very little improvement in PAPR when $r = 2$. Fig. 3 shows the CCDFs of PAPR of the reduced complexity PTS technique as a function of I and r for $N = 128$. It is shown that the trends are similar to those in Fig. 2. It can be concluded from Figs. 2 and 3 that a small number of iterations, i.e., 2 or 3, is sufficient for the convergence of the proposed technique. It is also shown in Figs. 2 and 3 that the the PAPR statistic is much better for $r = 2$ than for $r = 1$.

Fig. 4 shows the CCDFs of PAPR of various PTS techniques for $N = 64$. It is shown in Fig. 4 that the unmodified OFDM

[1]Although it is not shown here due to space limitations, we have found that the initial vector of phase factors does not affect the performance of the proposed technique.

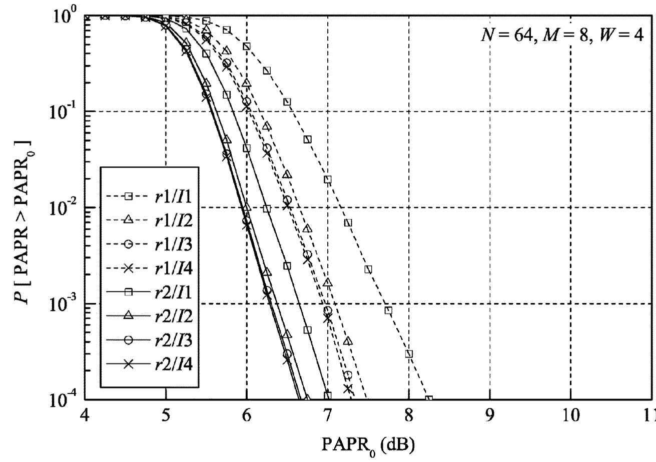

Fig. 2. CCDFs of PAPR of reduced complexity PTS technique with $N = 64, M = 8$, and $W = 4$ with $P = \{\pm 1, \pm j\}$.

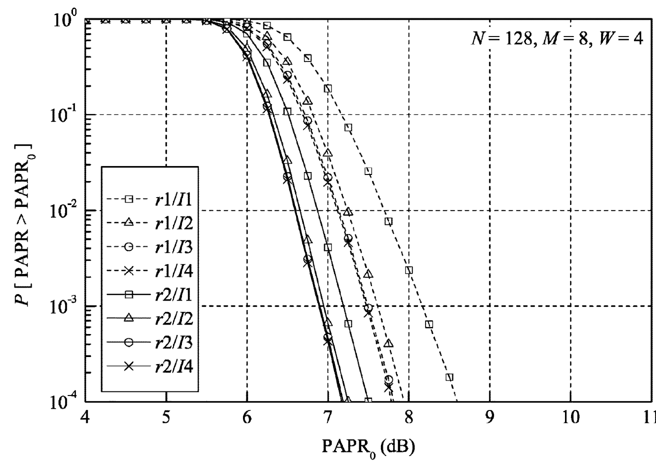

Fig. 3. CCDFs of PAPR of reduced complexity PTS technique with $N = 128, M = 8$ and $W = 4$ with $P = \{\pm 1, \pm j\}$.

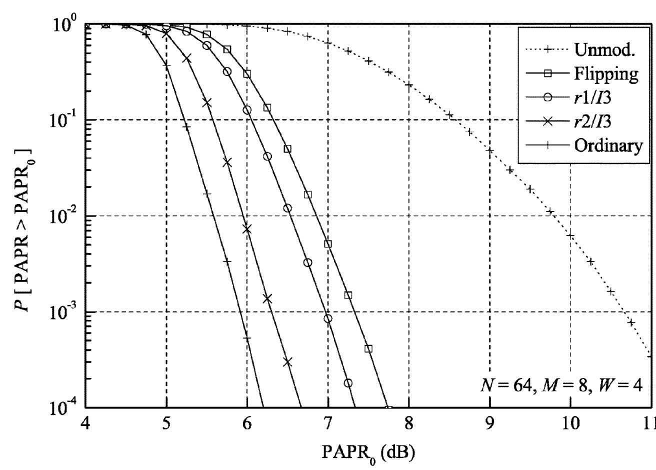

Fig. 4. CCDFs of PAPR of various PTS techniques with $N = 64, M = 8$, and $W = 4$ with $P = \{\pm 1, \pm j\}$.

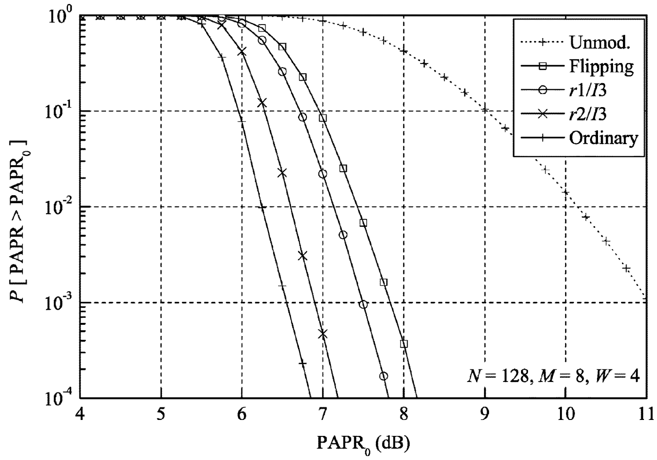

Fig. 5. CCDFs of PAPR of various PTS techniques with $N = 128, M = 8$, and $W = 4$ with $P = \{\pm 1, \pm j\}$.

signal (Unmod.) has a PAPR which exceeds 10.65 dB for less than 0.1% of the blocks. By using the ordinary PTS technique, 0.1% PAPR reduces to 5.90 dB. For the iterative flipping algorithm, the 0.1% PAPR is 7.35 dB. So, there is a gap of 1.45 dB between the 0.1% PAPR of the ordinary PTS technique and that of the iterative flipping algorithm. For the reduced complexity PTS technique, the 0.1% PAPR is 7.00 dB when $r = 1$ and $I = 3$ and is 6.30 dB when $r = 2$ and $I = 3$. Hence, the degradation in 0.1% PAPR, when compared with the ordinary PTS technique, is 1.10 dB for $r = 1$ and $I = 3$ and is only 0.4 dB for $r = 2$ and $I = 3$. Fig. 5 shows the CCDFs of PAPR of various PTS techniques for $N = 128$. It is shown in Fig. 5 that the 0.1% PAPR of the unmodified OFDM signal, the iterative flipping algorithm, the proposed technique with $r = 1$ and $I = 3$, the proposed technique with $r = 2$ and $I = 3$, and the ordinary PTS technique are 11.0, 7.80, 7.50, 6.90, and 6.55 dB, respectively. The results described above show that the reduced complexity PTS technique achieves better PAPR statistic than the iterative flipping algorithm and the amount of performance improvement increases with the increase in r.

The performance improvement of the proposed technique over the iterative flipping algorithm is due to the fact that it can go closer to the optimal phase factors than the iterative flipping algorithm in probabilistic sense due to the gradient descent search. The performance of the technique is better for larger r since larger number of vectors of phase factors are searched for larger r in every update of the phase factors.

We also compared the search complexity of the proposed technique with that of the ordinary PTS technique. When $N = 128$, $M = 8$, and $W = 4$ with $P = \{\pm 1, \pm j\}$, the search complexity of the proposed technique with $r = 1$ and $I = 3$ and that of the proposed technique with $r = 2$ and $I = 3$ are about 0.5% and 6% of the search complexity of the ordinary PTS technique.

V. Conclusion

In this letter, we proposed a reduced-complexity PTS technique for OFDM. In the proposed technique, gradient-descent search is performed to obtain the phase factors. The PAPR statistic of the proposed technique is much better than that of the iterative flipping algorithm and is very close to that of the ordinary PTS technique with significantly reduced search complexity. The proposed technique can be an alternative solution for reducing the complexity of the ordinary PTS technique with little performance degradation.

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