Modified Selected Mapping Technique for PAPR Reduction of Coded OFDM Signal

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***Abstract*—High peak-to-average power ratio (PAPR) of the transmitted signal is a major drawback of orthogonal frequency division multiplexing (OFDM). In this paper, we propose a modified selective mapping (SLM) technique for PAPR reduction of coded OFDM signal. In this technique, we embed the phase sequence, which is used to lower the PAPR of the data block, in the check symbols of the coded OFDM data block. It is shown that we can achieve both PAPR reduction from the SLM technique as well as error performance improvement from the channel coding with no loss in data rate from the transmission of side information. In addition, approximate expression for the complementary cumulative distribution function (CCDF) of the PAPR of the modified SLM technique is derived and compared with the simulation results.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has many well known advantages such as robustness in frequency-selective fading channels, high bandwidth efficiency, efficient implementation, and so on [1]. Hence, OFDM has made its way into many applications in both wireline and wireless environments. Some of well known examples include xDSL, digital audio broadcasting (DAB), digital video broadcasting-terrestrial (DVB-T), HIPERLAN/2, IEEE 802.11a, and IEEE 802.16. A major drawback of OFDM at the transmitter is the high peak-to-average power ratio (PAPR) of the transmitted signal. These large peaks require linear and consequently inefficient power amplifiers. To avoid operating the power amplifiers with extremely large back-offs, we must allow occasional saturation of the power amplifiers, resulting in in-band distortion and out-of-band radiation.

There are many solutions to reduce the PAPR of an OFDM signal. Some authors propose the use of block code, where the data sequence is embedded in a larger sequence and only a subset of all the possible sequences are used, specifically, those with low PAPR [2]. For example, the use of Golay complementary sequences [3] to reduce PAPR within 3 dB was proposed [4], [5]. Codes with both PAPR reduction and error correcting capability were also introduced in [6] by determining the relationship of the cosets of Reed-Muller codes to Golay complementary sequences. While block code reduces PAPR, it also reduces transmission rate, significantly so for a large number of subcarriers [7]. Furthermore, there is no effective coding technique with high code rate for a large number of subcarriers. Recently, multiple signal representation techniques have been proposed. These include partial transmit sequence (PTS) technique [8], selected mapping (SLM) technique [9], and interleaving technique [10]. These techniques improve PAPR statistics of an OFDM signal significantly without any in-band distortion and out-of-band radiation. But, they require side information to be transmitted from the transmitter to the receiver in order to let the receiver know what has been done in the transmitter. There are other approaches that do not require the transmission of side information. In one technique [11], a part of the subcarriers are used as peak reduction subcarriers and the value (amplitude and phase) of the peak reduction subcarriers are varied such that the resulting OFDM signal has lower PAPR. At the receiver, the information on the peak reduction subcarriers is simply ignored. But in this technique, a portion of subcarriers should be allocated as peak reduction subcarriers, resulting in a data rate loss.

To mitigate the performance degradation in the propagation channel, channel coding is usually used in communication systems [12], [13]. For OFDM, when channel coding is used it is possible to exploit frequency diversity in frequency-selective fading channels to obtain good performance under low signal-to-noise ratio conditions. Although many PAPR reduction techniques for OFDM have been proposed, techniques for reducing the PAPR of an OFDM signal with channel coding are yet to be developed. In this paper, we propose a modified SLM technique for the PAPR reduction of coded OFDM signal. The major advantage of the modified SLM technique is that there is no data rate loss from the transmission of the side information. Here, we present a phase sequence design method for coded OFDM signal, a method to embed the phase sequence on the check symbols of the coded OFDM data block, and a method to reliably recover the phase sequence at the receiver. It is shown that we can achieve both PAPR reduction from the SLM technique as well as error performance improvement from the channel coding with no loss in data rate from the transmission of side information. Also in this paper, approximate expression for the PAPR statistic of an OFDM signal after applying modified SLM technique is derived. It is shown that the approximate expression matches quite well with the simulation results.

The remainder of the paper is organized as follows. Section II defines the PAPR of an OFDM signal. In Section III, we briefly overview the SLM technique and channel coding; and then present the modified SLM technique for coded OFDM signal. Simulation results are presented and compared with approximations in Section IV. Finally, conclusions are drawn in Section V.

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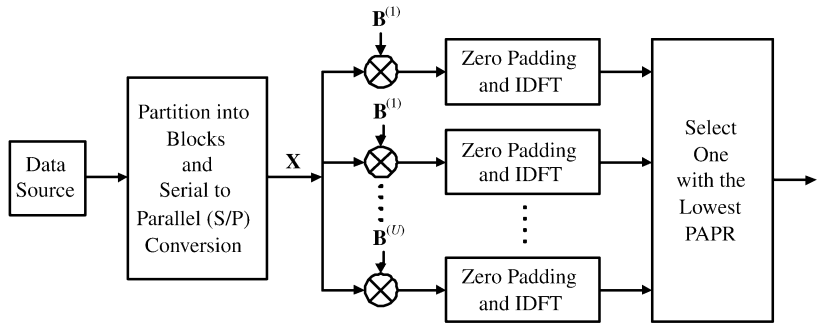

Fig. 1. Block diagram of OFDM transmitter with the SLM technique.

II. PAPR of an OFDM Signal

Let us denote the *data block* of length N as a vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ where N is equal to the number of subcarriers and $(\cdot)^T$ denotes transpose. The duration of a *data symbol* X_m in $\mathbf{X}$ is T. Each data symbol in $\mathbf{X}$ modulates one of a set of subcarriers, $\{f_m, m = 0, 1, \ldots, N-1\}$. The N subcarriers are chosen to be orthogonal, that is, $f_m = m\Delta f$, where $\Delta f = 1/NT$ and NT is the duration of an OFDM data block. The complex envelope of the transmitted OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m \cdot e^{j2\pi f_m t}, \quad 0 \leq t < NT. \tag{1}$$

The PAPR of the transmitted signal in (1) is defined as

$$\mathrm{PAPR} = \frac{\max\limits_{0 \leq t < NT} |x(t)|^2}{\frac{1}{NT} \cdot \int_0^{NT} |x(t)|^2 dt}. \tag{2}$$

In the remaining part of the paper, an approximation will be made in that only NL equidistant samples of (1) will be considered where L is an integer which is larger than or equal to 1. This 'L-times oversampled' time-domain signal samples are represented as a vector $\mathbf{x} = [x_0, x_1, \ldots, x_{NL-1}]^T$ and obtained as

$$\begin{aligned} x_k &= x\left(k \cdot \frac{T}{L}\right) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m \cdot e^{j(2\pi km/NL)}, \\ k &= 0, 1, \ldots, NL-1. \end{aligned} \tag{3}$$

It can be seen that the sequence $\{x_k\}$ can be interpreted as the inverse discrete Fourier transform (IDFT) of the OFDM data block $\mathbf{X}$ with $(L-1)N$ zero padding. It is well known that PAPR of the continuous-time OFDM signal cannot be obtained precisely by the use of the Nyquist rate sampling, which corresponds to the case of $L = 1$. It is shown in [14] that $L = 4$ can provide sufficiently accurate PAPR results. The PAPR computed from the L-times oversampled time-domain signal samples is given by

$$\mathrm{PAPR} = \frac{\max\limits_{0 \leq k < NL} |x_k|^2}{E\left[|x_k|^2\right]} \tag{4}$$

where $E[\cdot]$ denotes expectation.

III. Modified Selected Mapping Technique for Coded OFDM Signal

A. Review of SLM Technique

To begin with, we briefly review the ordinary SLM technique [9]. Block diagram of the SLM technique is shown in Fig. 1. At first, input data is partitioned into a data block $\mathbf{X}$ of length N. Then the OFDM data block is multiplied element by element with phase sequences $\mathbf{B}^{(u)} = [b_{u,0}, b_{u,1}, \ldots, b_{u,N-1}]^T$, $u = 1, 2, \ldots, U$, to make the U phase rotated OFDM data blocks $\mathbf{X}^{(u)} = [X_{u,0}, X_{u,1}, \ldots, X_{u,N-1}]^T$ where $X_{u,m} = X_m \cdot b_{u,m}$, $m = 0, 1, \ldots, N-1$. All U phase rotated OFDM data blocks represent the same information as the unmodified OFDM data block provided that the phase sequence is known. To include unmodified OFDM data block in the set of the phase rotated OFDM data blocks, we may set the first phase sequence $\mathbf{B}^{(1)}$ as all one vector of length N. After applying the SLM technique to $\mathbf{X}$, (1) becomes

$$\begin{aligned} x^{(u)}(t) &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m \cdot b_{u,m} \cdot e^{j2\pi f_m t}, \quad 0 \leq t < NT, \\ & u = 1, 2, \ldots, U. \end{aligned} \tag{5}$$

PAPR is calculated for U phase rotated OFDM data blocks by using (3) and (4). Among the phase rotated OFDM data blocks, one with the lowest PAPR is selected and transmitted. The information about the selected phase sequence should be transmitted to the receiver as side information. At the receiver, reverse operation should be performed to recover the unmodified OFDM data block.

SLM technique needs U IDFT operations for each OFDM data block and the number of required side information bits is $\lfloor \log_2 U \rfloor$ where $\lfloor y \rfloor$ denotes the smallest integer which does not exceed y. The phase sequences are selected in a way such that the phase rotated OFDM data blocks are 'sufficiently' different. In the ordinary SLM technique, there is no restriction on the construction of the phase sequences. However, we set a structural limitation on the phase sequences for the modified SLM technique described below.

B. Channel Coding for OFDM

Various channel coding techniques, such as block code, convolutional code, or Turbo code can be used for OFDM. Here, we consider a simple block code. A linear block code $\mathbb{C}$ is a nonempty set of n-tuples over $\mathrm{GF}(q)$, called codeword, such

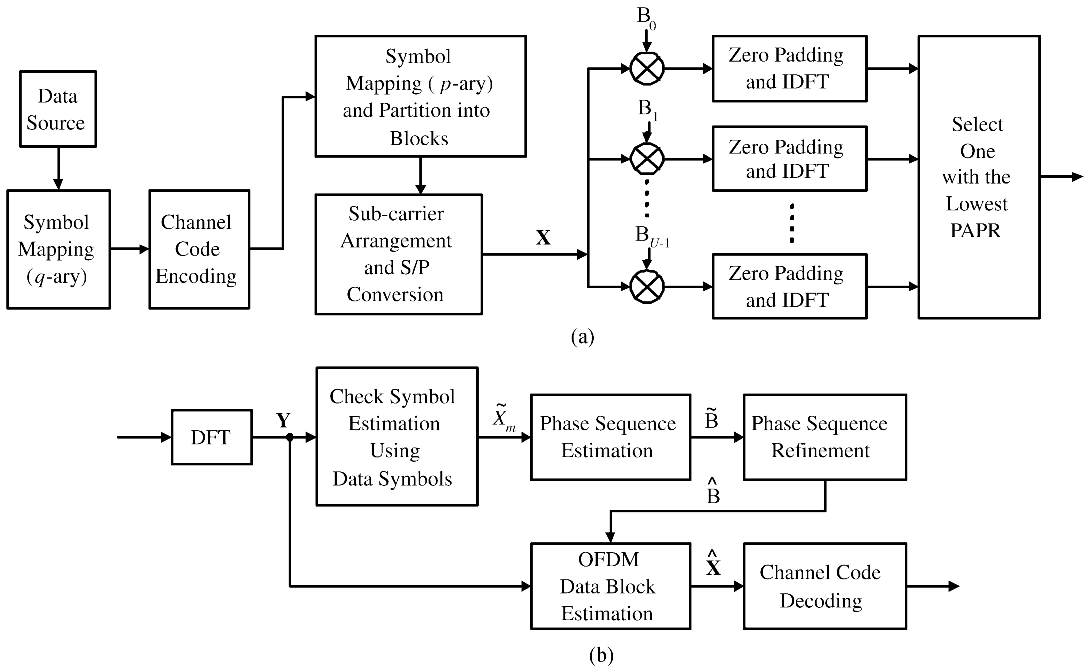

Fig. 2. Block diagram of the modified SLM technique. (a) Transmitter. (b) Receiver.

that the sum of two codewords is a codeword, and the product of any codeword with a field element is a codeword [12], [13]. Let k be the dimension of $\mathbb{C}$. The code $\mathbb{C}$ is referred to as an (n,k) code where n is the block length of the code and k is the dimension of code. When we are dealing with the linear block code, it is convenient to use systematic encoder. A systematic encoder for an (n,k) block code is one that maps each dataword (consists of k data symbols) into a codeword with the k data symbols unmodified in the first k symbols of the codeword. The remaining $(n-k)$ symbols are called as the check symbols. We may use a systematic convolutional code, such as rate 1/2 systematic convolutional code in which half of the symbols are data symbols and the rest half of the symbols are check symbols. It should be noted that channel coding is used for error correction, not for PAPR reduction in this paper. When the code rate is $r = k/n$, data rate is reduced by a factor of r due to the channel coding.

C. Modified SLM Technique for Coded OFDM Signal

Fig. 2 shows the block diagram of the modified SLM technique. At first, input data is mapped into q-ary symbols and then processed by a rate $r = k/n$ code over $\mathrm{GF}(q)$. After channel code encoding, data symbols and check symbols are separately mapped to p-ary symbols and are grouped into blocks of length N. The number of codewords required to make an OFDM data block of length N is given by

$$M = \frac{N \log_2 p}{n \log_2 q}. \tag{6}$$

After collecting M codewords, the data symbols and check symbols are arranged appropriately among N subcarriers. $N \times (k/n)$ subcarriers are used to transmit data symbols and the rest $N \times (n-k)/n$ subcarriers are used for check symbols. Fig. 3 shows an example of such subcarrier arrangement when $N = 10$, $p = q = 4$, $(n,k) = (5,3)$, and $M = 2$. In this example, 2 codewords of length 5 make an OFDM data block of length 10. The first 3 subcarriers are used to transmit the first dataword. The next 2 subcarriers are used to transmit the check symbols of the first codeword. The second codeword is mapped to the rest 5 subcarriers in a similar way. The specific carrier arrangement may vary according to system requirements. Note again that the check symbols are transmitted using separate subcarriers.

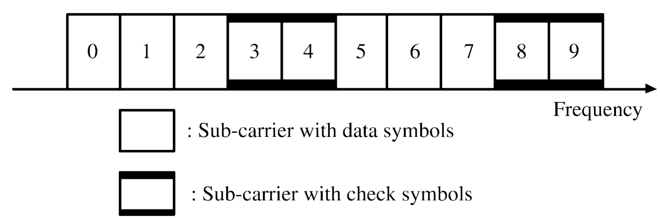

Fig. 3. Example of subcarrier arrangement for $N = 10$ with $(5,3)$ block code.

After making an OFDM data block, we need to construct U phase rotated OFDM data blocks for the SLM technique. Special attention should be paid to the design of the phase sequences used in the proposed technique. In the ordinary SLM technique, there is no restriction on the design of phase sequences as long as they are sufficiently different. In the proposed technique, however, we set the element of phase sequences, which correspond to the positions of data symbols, as *one*. In other words, the phase factors have a limitation on the positions of subcarriers in which the phase factors can have arbitrary phase. Hence, all U phase rotated OFDM blocks have same values in the positions of data symbols. In Fig. 3, these positions are 0, 1, 2, 5, 6, and 7. Other positions of the phase sequences may have arbitrary phase. An example of the set of phase sequences for $N = 10$, $p = q = 4$, with $(5,3)$ block code when $U = 4$ is shown in Fig. 4. As in the ordinary SLM technique, U

$\mathbf{b}_0$	1	1	1	1	1	1	1	1	1	1
$\mathbf{b}_1$	1	1	1	j	$-j$	1	1	1	-1	1
$\mathbf{b}_2$	1	1	1	-1	j	1	1	1	j	-1
$\mathbf{b}_3$	1	1	1	$-j$	1	1	1	1	$-j$	j

Fig. 4. Example of the set of phase sequences for $N = 10$ with $(5, 3)$ block code when $U = 4$.

phase rotated OFDM data blocks are generated using these phase sequences and one with the lowest PAPR is chosen and transmitted to the receiver. We will exploit the fact that 'the data symbols are not modified in all phase rotated OFDM data blocks' to estimate selected phase sequence with no data rate loss at the receiver.

At the receiver, the information on the selected phase sequence is required to recover transmitted OFDM data block from the received OFDM data block. In general, the set of all U phase sequences are known both to the transmitter and the receiver and the transmitter sends the index of the selected phase sequence as side information. In the ordinary SLM technique, the information on selected phase sequence should be explicitly contained in the OFDM data block, resulting in a data rate loss. In the proposed technique, however, the selected phase sequence itself is contained in the check symbols of the transmitted OFDM data block. We can extract the selected phase sequence from the received OFDM data block itself. Brief description on the estimation of the selected phase sequence is as follows:

- Using the data symbols in the received OFDM data block, the estimates for the check symbols are obtained.
- The estimate for each element of the selected phase sequence is obtained by dividing the received check symbol (which is phase rotated due to the phase sequence) by the estimated check symbol of the corresponding position.
- The estimate for the selected phase sequence is obtained by finding a phase sequence that is the closest to the phase sequence estimate among all possible candidates.

We will describe the phase sequence estimation process in detail. Let us denote the received OFDM data block after demodulation as $\mathbf{Y} = [Y_0, Y_1, \ldots, Y_{N-1}]^T$. For convenience, we assume that the structure of the OFDM data block is similar to the structure in Fig. 3. Note that this structure is valid when $p = q$ only. In this case, $Y_{i\cdot n}, Y_{i\cdot n+1}, \ldots, Y_{(i+1)\cdot n-1}$ constitute a received codeword for $i = 0, 1, \ldots, M-1$, where $M = N/n$. $Y_{i\cdot n}, Y_{i\cdot n+1}, \ldots, Y_{i\cdot n+k-1}$ are the received data symbols and the rest are the received check symbols. At first, by re-encoding the received data symbols, we make the estimate for the check symbols $\tilde{X}_{i\cdot n+k}, \tilde{X}_{i\cdot n+k+1}, \ldots, \tilde{X}_{(i+1)\cdot n-1}$, $i = 0, 1, \ldots, M-1$. Then, we can make an estimate for each element of the selected phase sequence by dividing the received check symbols by the estimate for the check symbols, symbol by symbol. The estimated phase sequence $\tilde{\mathbf{B}} = [\tilde{b}_0, \tilde{b}_1, \ldots, \tilde{b}_{N-1}]^T$ can be written as

$$\tilde{b}_m = \begin{cases} 1, & \text{for positions of data symbols,} \\ \frac{Y_m}{\tilde{X}_m}, & \text{for positions of check symbols.} \end{cases} \tag{7}$$

The estimated phase sequence may have errors in some positions due to various reasons. Therefore, with the knowledge of the set of U phase sequences in the receiver, we can refine the phase sequence estimate by finding a phase sequence with minimum Euclidian distance from $\tilde{\mathbf{B}}$ among the phase sequences $\mathbf{B}^{(u)}$, $u = 1, 2, \ldots, U$. The refined phase sequence estimate $\hat{\mathbf{B}} = [\hat{b}_0, \hat{b}_0, \ldots, \hat{b}_{N-1}]$ is given by

$$\hat{\mathbf{B}} = \arg\min_{\mathbf{B}\in\{\mathbf{B}^{(u)}, u=1,2,\ldots,U\}} \left\{ \left| \mathbf{B} - \tilde{\mathbf{B}} \right|^2 \right\}. \tag{8}$$

Now, we have refined estimate for the selected phase sequence. We obtain the estimate for unmodified OFDM data block before applying SLM technique using the refined estimate for the selected phase sequence. The estimate for the unmodified OFDM data block $\hat{\mathbf{X}} = [\hat{X}_0, \hat{X}_1, \ldots, \hat{X}_{N-1}]$ can be obtained as

$$\hat{X}_m = \begin{cases} Y_m, & \text{for positions of data symbols,} \\ \frac{Y_m}{\hat{b}_m}, & \text{for positions of check symbols.} \end{cases} \tag{9}$$

Finally, channel code decoding [12], [13] is done for each codeword in the received OFDM data block.

D. Approximate Expression for CCDF of PAPR

The distribution of the PAPR of an OFDM signal has been derived in [15]. From the central limit theorem, the real and imaginary part of the time-domain signal samples follow Gaussian distribution each with a mean of zero and a variance of 0.5 for an OFDM signal with a large number of subcarriers, i.e., 64 or higher. Hence the amplitude of an OFDM signal has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom and a mean of zero. The cumulative distribution function (CDF) of the amplitude of a signal sample is given by

$$F(z) = 1 - e^{-z}. \tag{10}$$

What we want to derive is a CDF of PAPR of an OFDM data block. Assuming that the signal samples are mutually independent and uncorrelated, the CDF of the PAPR of an OFDM data block is derived as

$$P(\text{PAPR} \le z) = F(z)^N = (1 - e^{-z})^N. \tag{11}$$

The assumption made above that the signal samples is mutually independent and uncorrelated is not true anymore when oversampling is applied. It is suggested in [15] that the PAPR of oversampled signal for N subcarriers is approximated by the distribution for αN subcarriers without oversampling where α is larger than 1. In other words, the effect of oversampling is approximated by adding a certain number of extra signal samples. The distribution of PAPR for oversampled signal is given by

$$P(\text{PAPR} \le z) \approx (1 - e^{-z})^{\alpha \cdot N}. \tag{12}$$

By extensive simulations, we have found that $\alpha = 2.3$ is a good approximation for 4-times oversampled OFDM signals.

Next, we derive an approximate expression for the PAPR of the SLM technique. If the phase rotated OFDM data blocks from U different branches in the SLM technique is independent and uncorrelated, the complementary CDF (CCDF) of the PAPR of an OFDM signal after applying SLM technique with U branches is derived as

$$\begin{aligned} P(\mathrm{PAPR}_{\mathrm{SLM}}(U) > z) &= \prod_{u=1}^{U} P(\mathrm{PAPR}_u > z) \\ &= \prod_{u=1}^{U} \{1 - P(\mathrm{PAPR}_u \le z)\} \\ &= \{1 - P(\mathrm{PAPR} \le z)\}^U \\ &\approx \left\{1 - (1 - e^{-z})^{\alpha \cdot N}\right\}^U \end{aligned} \tag{13}$$

where $\mathrm{PAPR}_{\mathrm{SLM}}(U)$ denotes the PAPR of the SLM technique with U branches and PAPR_u denotes the PAPR of the uth branch in the SLM technique. Equation (13) can be used for SLM technique with no structural limitation on the phase sequences. In the modified SLM technique, however, we lost some of the randomness in the SLM technique from the restriction on the structure of the phase sequences. We may represent the PAPR of the modified SLM technique for coded OFDM signal as

$$P(\mathrm{PAPR}_{\mathrm{SLM},r}(U) > z) \approx \left\{1 - (1 - e^{-z})^{\alpha \cdot N}\right\}^{\beta \cdot U} \tag{14}$$

where $\mathrm{PAPR}_{\mathrm{SLM},r}(U)$ denotes the PAPR of the SLM technique with U branches for coded OFDM signal with code rate r and β (between 0 and 1) represents the amount of loss in randomness. The CCDF of the modified SLM technique can be approximated by (14) with properly chosen β.

IV. Simulation Results and Discussions

We use computer simulations to evaluate the performance of the proposed PAPR reduction technique. To approximate the effect of nonlinear power amplifier in the transmitter, we adopt Rapp's model for amplitude conversion [15]. The relation between amplitude of the normalized input signal A and amplitude of the normalized output signal $g(A)$ of the nonlinear power amplifier is given by

$$g(A) = \frac{A}{(1 + A^{2p})^{1/(2p)}} \tag{15}$$

where p is a parameter that represent the nonlinear characteristic of the power amplifier. The power amplifier approaches linear amplifier as p gets larger. We choose $p = 3$ which is a good approximation of a general power amplifier [15]. The phase conversion of the power amplifier is neglected in this paper. Fig. 5 shows the input-output relation curve of the Rapp's power amplifier model when $p = 3$. The input signal is normalized by a normalization factor to appropriately fit the input signal into the desired range in the input-output relation curve. The normalized output signal is processed back into original scale before normalization. The amount of nonlinear distortion depends on the output back-off (OBO) which is defined as

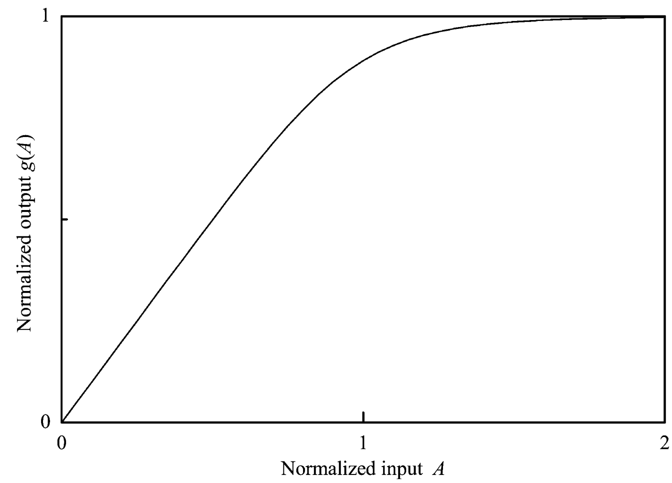

Fig. 5. Input–output relation curve of the Rapp's power amplifier model when $p = 3$.

$$\mathrm{OBO} = \frac{P_{o,\max}}{P_{o,\mathrm{avg}}} \tag{16}$$

where $P_{o,\max}$ is the output power at the saturation point and $P_{o,\mathrm{avg}}$ the average power of the output signal. As performance measures, we use the CCDF of the PAPR and symbol error rate (SER) in a Rayleigh fading channel. Modulation is QPSK for all subcarriers. Assume that each subcarrier experience identically independently distributed Rayleigh fading with additive white Gaussian noise. We set the oversampling factor $L = 4$. As a block coding scheme, we adopt a $(15, 7)$ Reed-Solomon (RS) code over $\mathrm{GF}(16)$ that has minimum distance of 9 and can correct up to 4 error in each codeword. We limit the value of the elements of the phase sequences in $\{1, -1, j, -j\}$.

Fig. 6 shows the PAPR of the proposed technique (Prop.) and ordinary SLM technique (Ord.). Fig. 6(a) shows the PAPR for $N = 120$ with $U = 8$, 16, 32. It is shown in Fig. 6(a) that the unmodified OFDM signal (designated as 'Unmodified' in the legend) has a PAPR which exceeds 10.9 dB for less than 0.1% of the OFDM data blocks for $N = 120$. In this case, we say that the 0.1% PAPR of the unmodified OFDM signal is 10.9 dB. When $U = 8$, the 0.1% PAPR of the ordinary SLM technique and that of the proposed technique is 8.10 dB and 7.45 dB, respectively. When $U = 16$, the 0.1% PAPR of the ordinary SLM technique and that of the proposed technique is 7.90 dB and 7.35 dB, respectively. When $U = 32$, the 0.1% PAPR of the ordinary SLM technique and that of the proposed technique is 7.60 dB and 7.10 dB, respectively. The performance difference between the proposed technique and the ordinary SLM technique is due to the fact that, in the proposed technique, the phase sequences have a limitation on their structure and thus the improvement of PAPR provided by the proposed technique is not so much as the that from the ordinary SLM technique. In fact, the difference in PAPR improvement between the proposed technique and the ordinary SLM technique is closely related to the value of n, k or the code rate $r = k/n$. If code rate is close to 0, there is little difference between the CCDF of the proposed technique

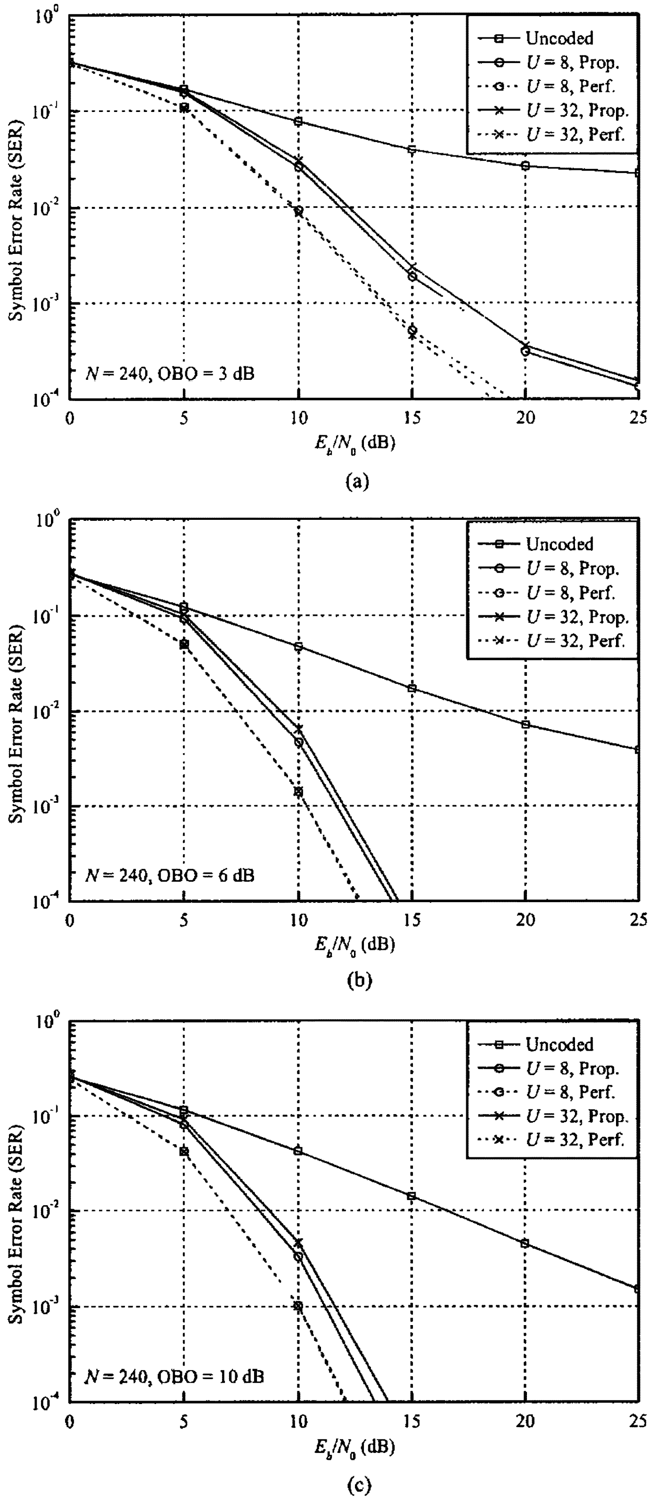

Fig. 6. SER of the modified SLM technique for $N = 240$. (a) SER for $\mathrm{OBO} = 3$ dB. (b) SER for $\mathrm{OBO} = 6$ dB. (c) SER for $\mathrm{OBO} = 10$ dB.

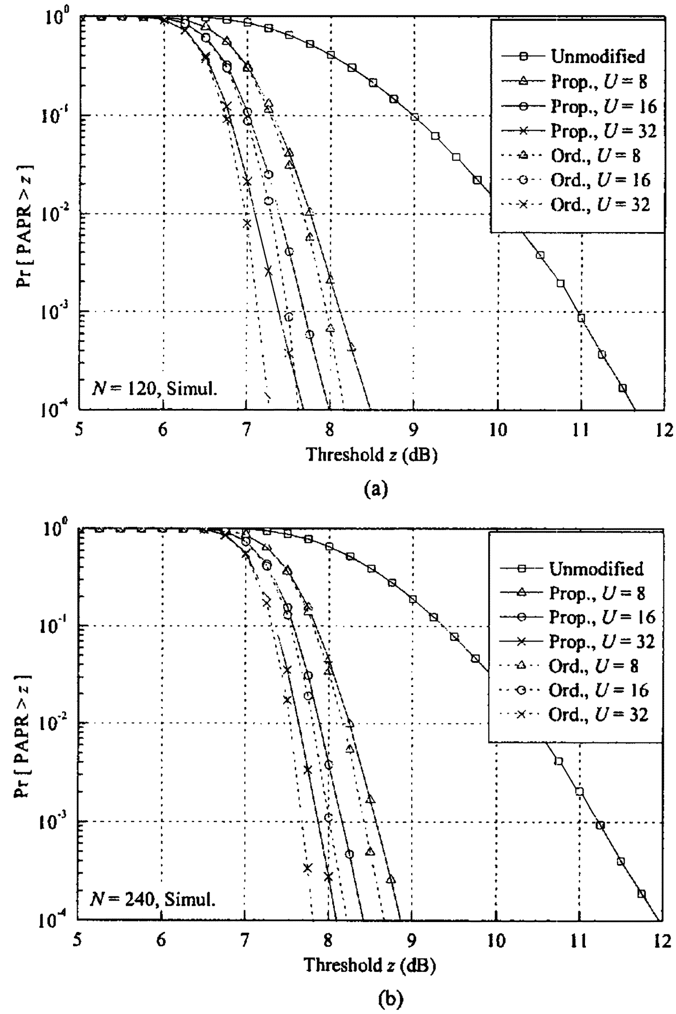

Fig. 7. PAPR of the modified SLM technique (simulation). (a) CCDFs of PAPR for $N = 120$. (b) CCDFs of PAPR for $N = 240$.

and that of the ordinary SLM technique. On the contrary, the higher the code rate is, the larger the difference is. But the difference remains quite small when compared with the difference between the proposed technique and unmodified OFDM signal if the code rate is lower than about a half. Fig. 6(b) shows the PAPR for $N = 240$ with $U = 8, 16, 32$. The trends are the similar for $N = 120$.

Fig. 7 plots the approximate expression for the CCDF of the ordinary SLM technique and that for the modified SLM technique. The approximate expression for the CCDF of the unmodified OFDM signal is also plotted in Fig. 7 for comparison. Fig. 7(a) shows the CCDF of PAPR for $N = 120$ and Fig. 7(b) shows that for $N = 240$. The CCDF of the unmodified OFDM signal is plotted by using (12) with $\alpha = 2.3$. The CCDF of the ordinary SLM technique is plotted by using (13) with $\alpha = 2.3$ and that of the modified SLM technique is plotted by using (14) with $\alpha = 2.3$ and $\beta = 0.80, 0.75, 0.70$ for $U = 8, 16, 32$, respectively. When we compare the CCDF plot of unmodified OFDM signal in Fig. 7 with the simulation results in Fig. 6, we can see that the approximate expression is quite accurate for $\alpha = 2.3$ when the oversampling factor is 4. We can also see that the approximate expression closely matches the simulation results for both the ordinary SLM technique and modified SLM technique with properly selected parameters α and β. The approximate expression can therefore be used to predict the PAPR statistic of the OFDM signal after applying SLM technique without simulations.

Fig. 8 shows the SER of the proposed technique for $N = 240$. SER of uncoded OFDM signal is also shown in Fig. 8 for comparison. In the legend, 'Prop.' refers to the proposed technique

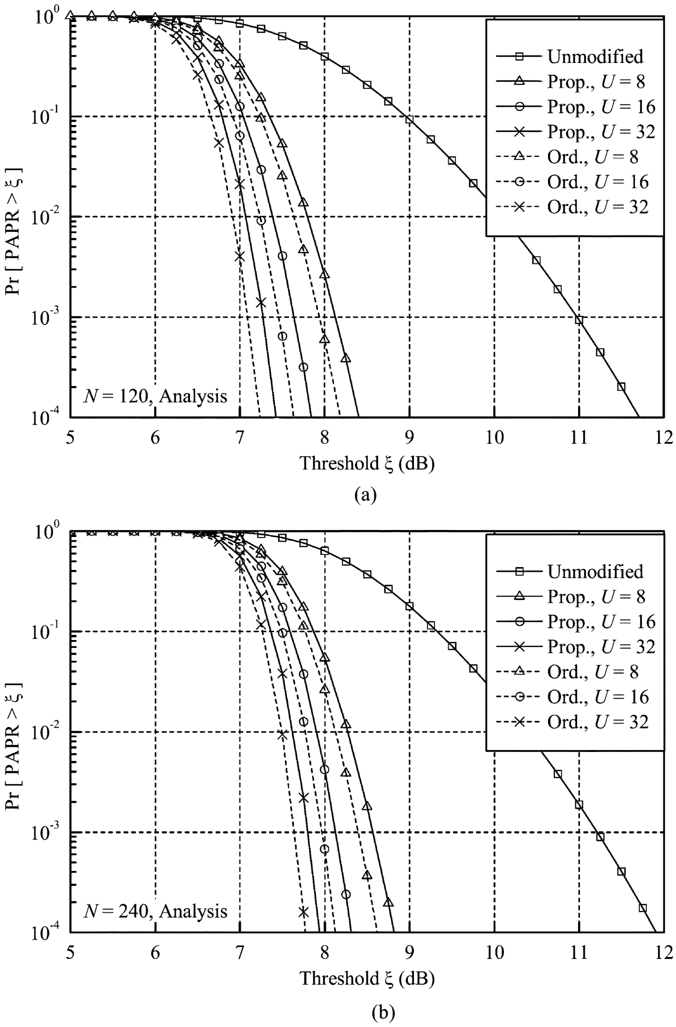

Fig. 8. PAPR of the modified SLM technique (analysis). (a) CCDFs of PAPR for $N = 120$. (b) CCDFs of PAPR for $N = 240$.

and 'Perf.' refers to the case when the phase sequence estimation at the receiver is perfect. Fig. 8(a)–(c) show the SER for $\mathrm{OBO} = 1$ dB, $\mathrm{OBO} = 3$ dB, $\mathrm{OBO} = 6$ dB, respectively. At first, we can see that SER performance is better for larger OBO values since the amount of nonlinear distortion is less for larger OBO. When the phase sequence estimation is perfect, the SER of $U = 32$ is slightly better than that for $U = 8$ since PAPR statistic is better for $U = 32$. On the contrary, it is shown that SER of the proposed technique is slightly better for $U = 8$ than for $U = 32$. This is due to that the minimum Euclidean distance among the set of U phase sequences is smaller for $U = 32$, resulting in a larger error in the estimation of the phase sequence at the receiver. It is also shown that when the phase sequence is estimated from the received OFDM data block in the proposed technique, there is some performance degradation when compared with the perfect estimation case. However, the performance degradation is not severe when compared with SER of the uncoded case. It can be concluded that most of the channel coding gain can be achieved by using the proposed technique.

During the final revision of this manuscript, we have found that a similar study was done independently in [16]. In [16], a modified PTS technique is proposed for coded OFDM signal. Some simulation results are provided to validate the technique; but analysis is not given for the PAPR statistic of an OFDM signal after applying the technique. It might be interesting to compare the proposed technique and the technique in [16] under a same framework.

V. Conclusions

In this paper, we proposed a modified SLM technique for the PAPR reduction of coded OFDM signal. By appropriately embedding the phase sequence information on the check symbols of the coded OFDM data block, we can achieve both PAPR reduction from the SLM technique and error performance improvement from the channel coding with no loss in data rate. We also derived approximate expression for the distribution of PAPR of modified SLM technique. It is shown that the approximate expression matches quite well with the simulation results with properly chosen parameters.

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