Channel Reliability Estimation for Turbo Decoding in Rayleigh Fading Channels With Imperfect Channel Estimates

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***Abstract*—In this letter we propose a simple estimation scheme of the channel reliability factor for turbo decoding in Rayleigh fading channels with imperfect channel estimates. The channel reliability factor is required for iterative MAP decoding of turbo codes and depends on the error variance of the channel estimate as well as the signal-to-noise ratio (SNR) when the channel estimation is imperfect. The proposed estimator is based on the statistical computations on the block observation of samples obtained by processing the received signal and provides an accurate estimate of the channel reliability factor for even a short block observation. Simulation results show that the use of this estimate in turbo decoding does not yield perceptible degradation against performance of the turbo decoder using the exact one.**

***Index Terms*—Channel estimation, channel reliability factor, iterative decoding, turbo codes.**

I. Introduction

ITERATIVE maximum *a posteriori* probability (MAP) decoding algorithm of turbo codes requires knowledge of the *channel reliability factor*, denoted by L_c, to optimally combine the systematic values with the extrinsic values obtained from previous iterations [3]. For AWGN and Rayleigh fading channels with perfect channel estimates, the channel reliability factor depends only on the channel SNR [2], [3]. On the other hand, it depends on the error variance of the channel estimate as well as the channel SNR when the channel estimation is not perfect [2]. Previous work related to the channel SNR estimation for turbo decoding can be found in [3]–[5]. However, none of work in the literature, except for [6], have treated an estimation scheme for L_c derived in [2], which includes the error variance of the channel estimate.

In this letter, we propose a simple estimation scheme for L_c given in [2] for iterative MAP decoding of turbo codes over Rayleigh fading channels when a channel estimator provides an unbiased channel estimate with a certain error variance. The proposed estimation scheme allows the turbo decoder to calculate the correct decoding metric in [2] for practical systems with channel estimation error.

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II. System Model

We consider a turbo coded system signaling through a frequency flat Rayleigh slow-fading channel using antipodal binary phase-shift keying (BPSK) modulation. The following discrete time system model is taken from [2]. Let $c_k \in \{-\sqrt{E_s}, +\sqrt{E_s}\}$, $1 \le k \le N/R_c$, be BPSK transmitted symbols, where N and R_c are the information block size and the code rate, respectively. In this case, the symbol energy is related to the bit energy by $E_s = E_b R_c$. Assuming perfect synchronization, the matched filter output at time k is given by

$$y_k = a_k \cdot c_k + n_k \tag{1}$$

where the channel coefficient a_k and the noise component n_k are zero-mean complex Gaussian random variables. The variances of a_k and n_k are defined as $E[|a_k|^2] = 2\sigma_a^2$ and $E[|n_k|^2] = 2\sigma_n^2 = N_0$.

We assume that the complex channel coefficients $\{a_k\}$ are independent and identically distributed (i.i.d.) as a consequence of the fully channel interleaving and the channel estimate h_k for a_k is modeled as [2]

$$h_k = a_k + m_k. \tag{2}$$

In (2), m_k represents the channel estimation error and is assumed to be a zero-mean complex Gaussian random variable independent of a_k. The variances of m_k and h_k are given by $E[|m_k|^2] = 2\sigma_m^2$ and $E[|h_k|^2] = 2\sigma_h^2 = 2\sigma_a^2 + 2\sigma_m^2$, respectively. Furthermore, the cross correlation coefficient between y_k and h_k is given by [2]

$$\mu_k \triangleq \frac{E[y_k h_k^*]}{\sqrt{E[|y_k|^2]E[|h_k|^2]}} = |\mu| \cdot \mathrm{sgn}(c_k). \tag{3}$$

The fading effect is compensated by the channel estimate h_k, yielding the decision variable as

$$z_k = y_k h_k^* = z_{k,r} + j z_{k,i} \tag{4}$$

where $j = \sqrt{-1}$, $z_{k,r}$ and $z_{k,i}$ denote the real and imaginary parts of z_k, respectively. Note that only $\{z_{k,r}\}$ are used by the turbo decoder due to the binary modulation [2]. The channel reliability factor required for iterative MAP decoding of turbo codes in this case is given by [2]

$$\begin{aligned} L_c &= \frac{2|\mu|}{\sigma_y \sigma_h (1 - |\mu|^2)} \\ &= \frac{4\sqrt{E_s}}{N_0}\, \sigma_a^2 \left\{ \sigma_m^2 \left(\frac{2E_s}{N_0} \sigma_a^2 + 1 \right) + \sigma_a^2 \right\}^{-1} \end{aligned} \tag{5}$$

TABLE I
PERFORMANCE OF THE L_c ESTIMATOR FOR VARIOUS VALUES OF THE TRUE L_c. $E_s = 1$, $\sigma_a^2 = 0.5$, $\sigma_m^2 = \alpha\sigma_n^2$, AND $L = 500$

α	$\bar{E}_s/N_0$, dB	L_c, dB	$E[\hat{L}_c]$, dB	NMSE$[\hat{L}_c]$, dB	-3dB Mismatch	+3dB Mismatch
0.0	-4.7712	1.2494	1.2642	-20.0445	0	0
	-3.7712	2.2494	2.2673	-20.9982	0	0
	-2.7712	3.2494	3.2633	-21.8846	0	0
	-1.7712	4.2494	4.2643	-22.6654	0	0
	0.2288	6.2494	6.2636	-23.8805	0	0
0.2	-3.7712	0.0051	0.0165	-18.7417	0	0
	-2.7712	1.2668	1.2782	-20.1349	0	0
	-1.7712	2.4864	2.5014	-21.2700	0	0
	0.2288	4.8201	4.8363	-23.0640	0	0
	1.2288	5.9438	5.9601	-23.6820	0	0
0.5	-2.7712	-0.6359	-0.6272	-18.0331	2	0
	-1.7712	0.7241	0.7324	-19.4987	0	0
	0.2288	3.2952	3.3048	-22.0581	0	0
	1.2288	4.5153	4.5290	-22.9100	0	0
	2.2288	5.6984	5.7156	-23.6437	0	0
1.0	-1.7712	-1.1957	-1.1892	-17.2079	7	0
	0.2288	1.5531	1.5704	-20.4373	0	0
	1.2288	2.8505	2.8681	-21.5230	0	0
	2.2288	4.1020	4.1151	-22.5296	0	0
	3.2288	5.3128	5.3288	-23.3237	0	0

which is a quantity to be estimated at the receiver because the decoder does not know it as the *a priori*.

III. ESTIMATION OF CHANNEL RELIABILITY FACTOR

Our objective is to device a blind algorithm which does not require the transmission of known training symbols to estimate the channel reliability factor L_c. To this end, we consider an estimator for L_c based on the statistical computations on the block observation of samples obtained by processing the received signal, similar to the scheme in [3]. We first express L_c in terms of the moments of y_k, h_k, and $|z_{k,r}|$.

The probability density function (pdf) of z_k conditioned on c_k is given by [2]

$$p(z_k|c_k) = \frac{1}{2\pi\sigma_y^2\sigma_h^2(1-|\mu|^2)} \exp\left\{\frac{\mu_k z_{k,r}}{\sigma_y\sigma_h(1-|\mu|^2)}\right\} \cdot K_0\left(\frac{|z_k|}{\sigma_y\sigma_h(1-|\mu|^2)}\right) \quad (6)$$

where $K_0(\cdot)$ is the modified Hankel function of the zeroth order. The integral representation of the modified Hankel function of the νth order is given by [8, eq. (8.432.6)]

$$K_\nu(z) = \frac{1}{2}\left(\frac{z}{2}\right)^\nu \int_0^\infty \frac{1}{t^{\nu+1}} \exp\left\{-t - \frac{z^2}{4t}\right\} dt \cdot |\arg z| < \pi/2,\ \mathrm{Re}\{z^2\} > 0. \quad (7)$$

From (6) and (7), the pdf of $z_{k,r}$ is given in (8), shown at the bottom of the page. Using the pdf (8), the nth moment of $|z_{k,r}|$ can be found as

$$\begin{aligned} E[|z_{k,r}|^n] &= \int_{-\infty}^{\infty} |z_{k,r}|^n \cdot p(z_{k,r})\, dz_{k,r} \\ &= \frac{n!}{2}\sigma_y^n\sigma_h^n\left\{(1-|\mu|)^{n+1} + (1+|\mu|)^{n+1}\right\}. \end{aligned} \quad (9)$$

From (9) with $n = 1$, the first moment of $|z_{k,r}|$ is given by

$$E[|z_{k,r}|] = \sigma_y\sigma_h\left(1+|\mu|^2\right). \quad (10)$$

Then, the channel reliability factor (5) can be expressed in terms of $E[|y_k|^2]$, $E[|h_k|^2]$, and $E[|z_{k,r}|]$ as follows:

$$L_c = \frac{2\sqrt{2E[|z_{k,r}|]\Big/\sqrt{E[|y_k|^2]\cdot E[|h_k|^2]} - 1}}{\sqrt{E[|y_k|^2]\cdot E[|h_k|^2]} - E[|z_{k,r}|]}. \quad (11)$$

As the samples y_k, h_k, and $z_{k,r}$ are observable at the receiver, we replace the expectations in (11) with the corresponding block averages in order to obtain an estimate for L_c, as in [3]. For notational convenience we have made the definition of the block average as $\langle x\rangle_L \triangleq L^{-1}\sum_{k=1}^L x_k$, where L is the number of samples used for averaging. Using the block averages of $|y_k|^2$, $|h_k|^2$ and $|z_{k,r}|$, we can obtain the estimate for L_c as follows:

$$\hat{L}_c = \frac{2\sqrt{2\langle|z_r|\rangle_L\Big/\sqrt{\langle|y|^2\rangle_L\cdot\langle|h|^2\rangle_L} - 1}}{\sqrt{\langle|y|^2\rangle_L\cdot\langle|h|^2\rangle_L} - \langle|z_r|\rangle_L} \quad (12)$$

$$\begin{aligned} p(z_{k,r}) &= \tfrac{1}{2}\int_{-\infty}^{\infty}\left\{p\left(z_k|c_k = +\sqrt{E_s}\right) + p\left(z_k|c_k = -\sqrt{E_s}\right)\right\} dz_{k,i} \\ &= \frac{1}{2\pi\sigma_y^2\sigma_h^2(1-|\mu|^2)}\cosh\left(\frac{|\mu|z_{k,r}}{\sigma_y\sigma_h(1-|\mu|^2)}\right)\int_{-\infty}^{\infty} K_0\left(\frac{|z_k|}{\sigma_y\sigma_h(1-|\mu|^2)}\right) dz_{k,i} \\ &= \frac{1}{2\pi\sigma_y^2\sigma_h^2(1-|\mu|^2)}\cosh\left(\frac{|\mu|z_{k,r}}{\sigma_y\sigma_h(1-|\mu|^2)}\right)\int_0^\infty\int_0^\infty \frac{1}{t}\exp\left\{-t - \frac{1}{4t}\frac{z_{k,r}^2 + z_{k,i}^2}{\sigma_y^2\sigma_h^2(1-|\mu|^2)^2}\right\} dz_{k,i}\, dt \\ &= \frac{1}{2\sigma_y\sigma_h}\cosh\left(\frac{|\mu|z_{k,r}}{\sigma_y\sigma_h(1-|\mu|^2)}\right)\exp\left\{\frac{|z_{k,r}|}{\sigma_y\sigma_h(1-|\mu|^2)}\right\} \end{aligned} \quad (8)$$

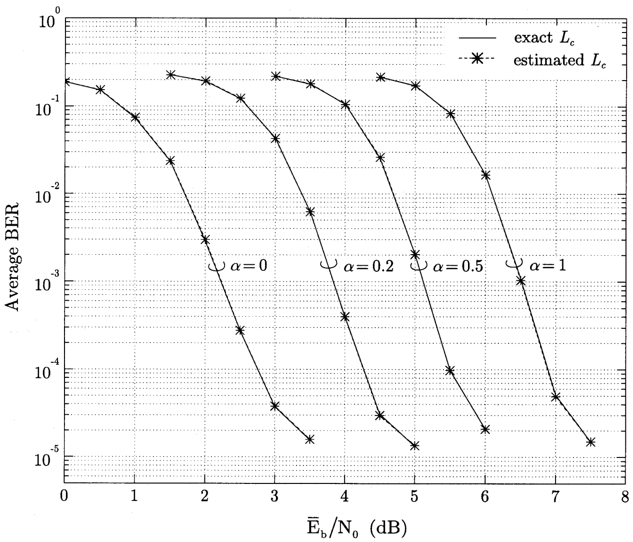

Fig. 1. Performance of the turbo decoder using the exact and estimated L_c for $\sigma_m^2 = \alpha\sigma_n^2$ with $\alpha = 0$, 0.2, 0.5, and 1. $N = 420$, $L = 500$, and the number of decoding iterations is eight.

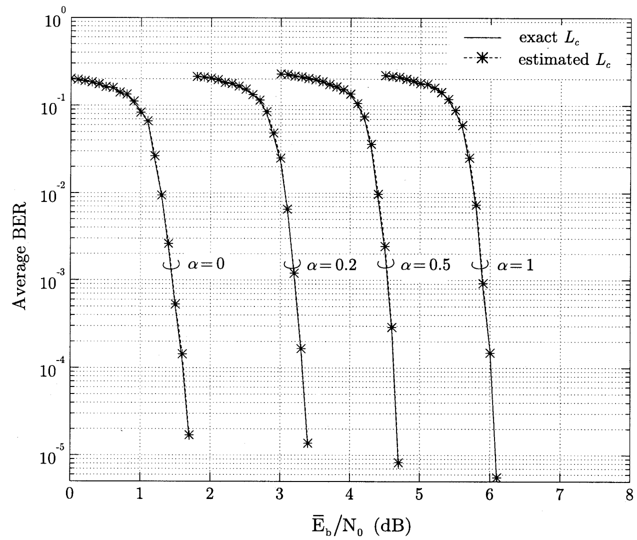

Fig. 2. Performance of the turbo decoder using the exact and estimated L_c for $\sigma_m^2 = \alpha\sigma_n^2$ with $\alpha = 0$, 0.2, 0.5, and 1. $N = 4096$, $L = 500$, and the number of decoding iterations is eight.

where $L = N/R_c$ if the samples of the entire block are used. To reduce computation complexity and latency, a smaller sample set rather than the entire block can be also used to estimate L_c.

To assess the accuracy of the proposed estimator, we evaluate the mean and the normalized mean square error (MSE)[1] of the estimates $\hat{L}_c$ for an observation length L of 500 code symbols. The results are shown in Table I. The symbol energy E_s and the fading power $2\sigma_a^2$ are normalized to one. Furthermore, we assume that $\sigma_m^2 = \alpha\sigma_n^2$, where α is a constant depending on the channel estimation algorithm. This assumption is due to the fact that most channel estimation algorithms produce better estimates when the SNR is high [2]. In Table I, for various values of the average symbol SNR[2] and α, the true L_c values are calculated by using (5) and the mean and the normalized MSE of the estimates obtained through the proposed estimator determined by over 20 000 trials. In addition, the number of poor estimates of greater than 3 dB above or below the true L_c value is also counted. Note that $\overline{E}_s/N_0$ values in the range of -4.7712 to 3.2288 dB correspond to the values of the average bit SNR, $\overline{E}_b/N_0$, in the range of 0–7 dB for the rate-1/3 code. From Table I, we can see that the mean values of the estimates $\hat{L}_c$ are quite close to the true values of L_c and the normalized MSE's are very small, even for a lower L_c. Also, the poor estimates do not occur except for the values of L_c far below the operating point of the turbo decoder.

Figs. 1 and 2 show simulation results for the BER performance of the turbo decoder using the proposed estimator to provide $\hat{L}_c$ for information block lengths N of 420 and 4096 bits, respectively. In these simulations, we considered a rate-1/3 turbo code employing two identical 8-state recursive systematic convolutional (RSC) encoders with generator $(13/15)_8$, separated by a random interleaver of length N. Irrespective of the entire block size, only 500 symbol observations were utilized to estimate L_c. We observe that there exists no perceptible degradation against the BER performance of the turbo decoder using the exact value of L_c. In [6], we provided similar results for the turbo coded pilot symbol assisted modulation (PSAM) system in correlated Rayleigh fading channel.

[1]As in [7], the normalized MSE is used as a performance measure of the estimator in that it reflects both the bias and the variance of an estimate. The normalized MSE of L_c is defined as $\mathrm{NMSE}[\hat{L}_c] \triangleq L_c^{-2}E[(\hat{L}_c - L_c)^2]$.

[2]The average SNR per symbol is defined as $\overline{E}_s/N_0 \triangleq E[|a_k|^2](E_s/N_0)$.

IV. Conclusions

In this letter we proposed an in-service estimation scheme of the channel reliability factor required iterative MAP decoding of turbo codes over Rayleigh fading channels when the channel estimation is imperfect. The proposed scheme estimates the channel reliability factor from the statistical computations on the block observation of matched filter outputs, channel estimates, and the decoder inputs, prior to decoding.

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