Abstract - In this paper, adaptive multi-stage parallel interference cancellation (PIC) receiver is considered for a multi-rate DS-CDMA system. In each stage of the adaptive multi-stage PIC receiver, multiple access interference (MAI) estimates are obtained using the sub-bit estimates from the previous stage and the adaptive weights for the sub-bit estimates. The adaptive weights are obtained by minimizing the mean squared error between the received signal and its estimate through normalized least mean square (LMS) algorithm. It is shown that the adaptive multi-stage PIC receiver achieves smaller BER than the matched filter receiver, multi-stage PIC receiver, and multi-stage partial PIC receiver for the multi-rate DS-CDMA system in a Rayleigh fading channel.

I. INTRODUCTION

Wireless communication systems are to support voice, data, and video services which demand multi-rate transmission [1]. To implement multi-rate transmission in a direct-sequence code division multiple-access (DS-CDMA) system, the multiple processing gain (MPG) scheme is used [2]. In the MPG scheme, a user with high data rate has smaller processing gain and a user with low data rate has larger processing gain while their chip rates are the same. The signal of a high-rate user has larger amplitude than that of a low-rate user so that the former has same bit energy as the latter while the former has shorter bit duration than the latter.

The performance of a multi-rate DS-CDMA system is severely degraded by the multiple access interference (MAI), the difference of transmitted power between users, and the near-far effect. Multi-user detectors are adopted to mitigate the performance degradation from MAI [3]-[5]. As a multi-user detector, a multi-stage parallel interference cancellation (PIC) receiver achieves performance improvement with moderate complexity and delay [6].

However, in the multi-stage PIC receiver the performance is not always improved as the number of stages increases, especially when the number of users approaches the processing gain of the DS-CDMA system. To improve the performance of the multi-stage PIC receiver, Divsalar et al. proposed a partial cancellation of the estimate for the MAI by introducing a partial cancellation factor in each stage [7]. Xue et al. proposed an adaptive multi-stage PIC receiver for a DS-CDMA system. In each stage of the adaptive multi-stage PIC receiver, MAI estimates are obtained using the bit estimates from the previous stage and the adaptive weights for the bit estimates [8]. The adaptive weights are obtained by minimizing the mean squared error between the received signal and its estimate. It is shown in [8] that the adaptive multi-stage PIC receiver has lower bit error rate (BER) than the matched filter (MF) receiver and multi-stage partial PIC receiver for the DS-CDMA system.

In this paper, the adaptive multi-stage PIC receiver is considered for the multi-rate DS-CDMA system with the MPG scheme and its bit error rate (BER) is obtained for a Rayleigh fading channel by simulation.

II. SYSTEM MODEL

Consider an asynchronous multi-rate DS-CDMA system. Suppose that the MPG scheme is adopted to accommodate multi-rate users which are divided into $N$ groups for their data rates. Let the number of users, the transmission rate of users, and the bit duration of the users in the group $n$ be denoted by $K_n$, $R_n$, and $T_n$, $n=1,2,\ldots,N$, respectively. $R_n$ is an integer multiple of $R_1$, i.e., $R_n = M_n R_1$, $n=1,2,\ldots,N$, with restriction that $1=M_1<M_2<\cdots<M_N$ and $M_n$ divides $M_N$, $n=1,2,\ldots,N$. During the bit duration of a group 1 user $T_1$, a group $n$ user transmits $M_n$ bits with bit duration $T_n = T_1 / M_n$. Assume that chip duration $T_1$ is the same for users in all groups. Then the processing gain of a group $n$ user becomes $L_n = T_n / T_1 = L_1 / M_n$.

The baseband signal transmitted by the user $k$ in the group $n$ is given by

$$s_{k,n}(t) = P_n(t) \sum_m b_{k,n}[m] a_{k,n}(t - mT_n)$$  \hspace{1cm} (1)$$

where $P_n(t)$ is the transmitted signal amplitude of a group $n$ user, $b_{k,n}[m] \in \{-1,1\}$ is the $m$-th source bit of the user $k$ in the group $n$, and $a_{k,n}(t)$ is the signature waveform of the user $k$ in the group $n$. The signature waveform of the user $k$ in the group $n$ is given by

$$a_{k,n}(t) = \sum_{l=0}^{L_n-1} a_{k,n}[l] p_{L_n}(t - lT_n)$$  \hspace{1cm} (2)$$

This work was supported in part by the Korean Ministry of Information and Communications.
where \( a_{k,n}[t] \in \{-1,1\} \) is the \( l \)-th chip of the signature waveform of the user \( k \) in the group \( n \), \( 0 \leq l \leq L_n - 1 \), and \( p_{g}(t) \) is a unit amplitude rectangular pulse with duration \( T_g \). The signal transmitted by the user \( k \) in the group \( n \) propagates over a channel having fading of \( a_{k,n}(t) \). The received baseband signal from the user \( k \) in the group \( n \) is given by

\[
x_{k,n}(t) = a_{k,n}(t)s_{k,n}(t - \tau_{k,n})
\]

\[
= A_{k,n}(t)\sum_{m} b_{k,n}[m]a_{k,n}(t - mT_u - \tau_{k,n})
\]

where \( \tau_{k,n} \) and \( A_{k,n}(t) = a_{k,n}(t)p_{g}(t - \tau_{k,n}) \) are the delay and received signal amplitude of the user \( k \) in the group \( n \). The baseband signal received at the receiver is given by

\[
r(t) = \sum_{n=1}^{N} \sum_{l=1}^{L_n} x_{k,n}(t) + n(t)
\]

\[
= \sum_{n=1}^{N} \sum_{l=1}^{L_n} A_{k,n}(t)\sum_{m} b_{k,n}[m]a_{k,n}(t - mT_u - \tau_{k,n}) + n(t)
\]

where \( n(t) \) is an additive white Gaussian noise (AWGN) with the power spectral density of \( N_0/2 \) W/Hz.

At the matched filter (MF) output for the user \( k \) in the group \( n \), the decision statistic for the \( m \)-th bit is given by

\[
\hat{b}_{k,n}[m] = \int_{t_{k,n} + \tau_{k,n}}^{t_{k,n} + \tau_{k,n} + T_u} r(t) a_{k,n}(t - mT_u - \tau_{k,n}) dt.
\]

For the user \( k \) in the group \( n \), the \( m \)-th bit estimate is given by

\[
\hat{b}_{k,n}[m] = \text{sgn}(\hat{b}_{k,n}[m])
\]

where \( \text{sgn}(\cdot) \) is the signum function defined as

\[
\text{sgn}(u) = \begin{cases} 
1, & u \geq 0, \\
-1, & u < 0.
\end{cases}
\]

### III. ADAPTIVE MULTI-STAGE PIC RECEIVER FOR MULTI-RATE DS-CDMA SYSTEM

In the adaptive multi-stage PIC receiver for the multi-rate DS-CDMA system with the MPG scheme, a bit of each user is divided into sub-bits whose bit duration equals the bit duration of the highest-rate user \( T_N \). For a highest-rate user, a bit corresponds to a sub-bit. Fig. 1 shows the block diagram of the \( s \)-th stage of the adaptive multi-stage PIC receiver for the multi-rate DS-CDMA system with the MPG scheme. Initial sub-bit estimates for the adaptive multi-stage PIC receiver are obtained by the MF bank which is referred to as the stage 0. For the user \( k \) in the group \( n \) in the stage 0, the decision statistic for the \( i \)-th sub-bit \( d_{k,n}^{(i)}[i] \) is given by

\[
\hat{d}_{k,n}^{(i)}[i] = \int_{t_{k,n} + \tau_{k,n}}^{t_{k,n} + \tau_{k,n} + T_u} r(t) a_{k,n}(t - \left\lfloor \frac{M_n}{M_N} \right\rfloor T_u - \tau_{k,n}) dt
\]

where \( \lfloor x \rfloor \) is the greatest integer which does not exceed \( x \).

In the stage 0, the \( i \)-th sub-bit estimate for the user \( k \) in the group \( n \) is given by

\[
\hat{A}_{k,n}^{(i)}[i] = \text{sgn}(\hat{d}_{k,n}^{(i)}[i]).
\]

Assume that the sub-bit estimates for all users are obtained in the \((s-1)\)-th stage. In the \( s \)-th stage, \( 1 \leq s < S \), the sub-bit estimates for the user \( k \) in the group \( n \) are resampled by the signature sequence \( a_{k,n}(t) \) and multiplied by the estimate for signal amplitude \( A_{k,n}(t) \) and adaptive weights to produce an estimate for the received signal \( \hat{x}_{k,n}^{(s)}(t) \). In the \( s \)-th stage, the estimate for the received signal of the user \( k \) in the group \( n \) is given by

\[
\hat{x}_{k,n}^{(s)}(t) = A_{k,n}(t) \sum_{i} \hat{A}_{k,n}^{(i)}[i] \hat{d}_{k,n}^{(s-1)}[i] a_{k,n}(t - \left\lfloor \frac{M_n}{M_N} \right\rfloor T_u - \tau_{k,n})
\]

\[
\times p_{g}(t - iT_N - \tau_{k,n})
\]

where \( p_{g}(t) \) is a unit amplitude rectangular pulse with duration \( T_N \) and \( \hat{d}_{k,n}^{(s-1)}[i] \) is the adaptive weight for the \( i \)-th sub-bit of the user \( k \) in the group \( n \).

To obtain the adaptive weights, the received signal is sampled at the chip-rate. Suppose that the delay of each user is an integer multiple of a chip interval, i.e., \( \tau_{k,n} = \xi_{k,n} T_c \) for some integer \( \xi_{k,n} \), \( k = 1,2,\cdots,K_n \), \( n = 1,2,\cdots,N \). The received signal after chip-rate sampling is given by

\[
r[j] = \sum_{n=1}^{N} \sum_{k=1}^{K_n} A_{k,n}(jT_c) b_{k,n}\left[ \left( j - \xi_{k,n} \right)/L_n \right] \]

\[
\times a_{k,n}\left[ \left( j - \xi_{k,n} \right)/L_n \right] + n[j]
\]

where \( n[j] \) is the noise sample. In the \( s \)-th stage, the estimate for the received signal after chip-rate sampling is given by

\[
\hat{r}^{(s)}[j] = \sum_{n=1}^{N} \sum_{k=1}^{K_n} \hat{A}_{k,n}(jT_c) \hat{d}_{k,n}^{(s-1)}\left[ \left( j - \xi_{k,n} \right)/L_n \right] \]

\[
\times a_{k,n}\left[ \left( j - \xi_{k,n} \right)/L_n \right]
\]

The adaptive weights are obtained by minimizing the mean squared error between the received signal and its estimate, i.e., the adaptive weights should satisfy

\[
\min E\left[ \left\| r[j] - \hat{r}^{(s)}[j] \right\|^2 \right].
\]

The adaptive weights are adjusted via a normalized least mean square (LMS) algorithm that operates in a sub-bit interval and on a chip basis. For the \( i \)-th sub-bit of the user \( k \) in the group \( n \) in the \( s \)-th stage, the equation for updating the adaptive weight \( \hat{A}_{k,n}^{(i)}[i] \) is given by
where $\lambda_{n,a}^{(i)}[i][l+1] = \lambda_{n,a}^{(i)}[i][l] + \sum_{i=1}^{N} \mu^{(i)} \lambda_{n,a}^{(i)}[i]\tilde{d}_{k,a}^{(i)}[i] - \sum_{i=1}^{N} K_i \times a_{k,a} ((iL_n + l - \xi_{k,a}) \bmod L_n) e^{(i)}[iL_n + l - \xi_{k,a}]$, $l = 0, 1, \ldots, L_n - 2$ \hspace{1cm} (14)

where $\lambda_{n,a}^{(i)}[i][l]$ is the adaptive weight after $l$-th iteration with $\lambda_{n,a}^{(i)}[i][0] = 1$, $e^{(i)}[j] = r(j) - \tilde{r}^{(i)}[j]$ is the error between the received signal and its estimate, and $\mu^{(i)}$ is the step size of the LMS algorithm for the $s$-th stage. $\lambda_{n,a}^{(i)}[i][L_n - 1]$ is used as the adaptive weight $\lambda_{n,a}^{(i)}[i]$ for the $i$-th sub-bit of the user $k$ in the group $n$ in the $s$-th stage. For LMS algorithm to converge, $\mu^{(i)}$ has to satisfy $0 < \mu^{(i)} < 2$. Larger step size result in faster convergence, but it causes higher gradient noise due to the misadjustment of the weights.

The partial summer sums up the estimates for the received signal of all users but the user $k$ in the group $n$ to obtain the MAI estimate for the user $k$ in the group $n$. For each user the MAI estimate is subtracted from the received signal and its result is passed on to the MF bank of the signal of all users but the user $k$. The sub-bit estimates are updated in each stage using the sub-bit estimate for the user $k$ in the group $n$, in the $s$-th stage. For LMS algorithm to converge, $\mu^{(i)}$ has to satisfy $0 < \mu^{(i)} < 2$. Larger step size result in faster convergence, but it causes higher gradient noise due to the misadjustment of the weights.

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In the $s$-th stage, the decision statistic for the $i$-th sub-bit is given by

$$d_{k,a}^{(i)}[l] = r(t) - \sum_{(i,h) \neq (i,a)} \tilde{x}_{h,a}^{(i)}(l).$$ \hspace{1cm} (15)

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$$d_{k,a}^{(i)}[l] = r(t) - \sum_{(i,h) \neq (i,a)} \tilde{x}_{h,a}^{(i)}(l).$$ \hspace{1cm} (16)

The $i$-th sub-bit estimate for the user $k$ in the group $n$ is given by

$$\hat{d}_{k,a}^{(i)}[l] = \text{sgn}(\tilde{d}_{k,a}^{(i)}[l]).$$ \hspace{1cm} (17)

The sub-bit estimates are updated in each stage using the sub-bit estimates from the previous stage.

In the $S$-th stage of the multi-stage PIC receiver, the $m$-th bit estimate for the user $k$ in the group $n$ is obtained as

$$\hat{b}_{k,a}[m] = \text{sgn} \left( \sum_{i=m(M_y/M_z) + 1}^{M_z} \sum_{i=m(M_y/M_z) + 1}^{M_z} \lambda_{k,a}^{(S)}[i][m] \tilde{d}_{k,a}^{(S)}[i] \right).$$ \hspace{1cm} (18)

IV. SIMULATION RESULTS

Consider an asynchronous multi-rate DS-CDMA system having the MPG scheme with three data rates: high-rate(HR), medium-rate(MR), and low-rate(LR). Suppose that BPSK is used as a modulation scheme, the Gold code of length 31 is used as a signature sequence for each user, and the Gold code is repeated once, 4, and 16 times during a bit interval of the HR, MR, and LR users. Assume that the channel for each user has independent identically distributed frequency-flat Rayleigh fading with normalized Doppler frequency $f_d T_s = 0.01$ where $T_s$ is the bit duration of the low-rate user. Assume that the system is chip-synchronous, each interfering user’s propagation delay is an integer multiple of a chip interval with respect to the delay of the desired user. Also assume that the receiver knows the signature sequences of all users and has perfect synchronization for each user and perfect power control for slow fading and path-loss. In each stage, the amplitude of the received signal of each user is estimated from the decision statistics in the previous stage.

Fig. 2 shows the BER of adaptive 2- and 3-stage PIC receiver with $\mu^{(1)} = 1.0$, $\mu^{(2)} = 0.5$, $\mu^{(3)} = 0.2$ in a Rayleigh fading channel for 8 LR, 4 MR, and 2 HR users. The BER of the conventional MF receiver, conventional 2- and 3-stage PIC receiver, 2- and 3-stage partial PIC receiver is also shown in Fig. 2 for comparison.

Fig. 2 a) shows the BER of the LR users. In Fig. 2 a) it is shown that various multi-stage PIC receivers achieve much smaller BER than the MF receiver. In Fig. 2 a) it is also shown that adaptive 2-stage PIC receiver achieves smaller BER than the 2-stage PIC receiver and the 2-stage partial PIC receiver. In Fig. 2 a) it is shown that the adaptive 3-stage PIC receiver has smaller BER than other 3-stage receivers for LR users. Fig. 2 b) and Fig. 2 c) show the BER of the MR users and HR users. In Fig. 2 b) and Fig. 2 c) it is shown that adaptive 2-stage PIC receiver achieves smaller BER than the 2-stage PIC receiver and the 2-stage partial PIC receiver for MR users and HR users. It is shown in Fig. 2 b) and Fig. 2 c) that the adaptive 3-stage PIC receiver has smaller BER than other 3-stage receivers for MR users and HR users. In Fig. 2 a), Fig. 2 b), and Fig. 2 c) it is also shown that the adaptive 2-stage PIC receiver achieves almost same BER as the 3-stage partial PIC receiver for all users with various data rates.

V. CONCLUSION

In this paper, adaptive multi-stage PIC receiver is examined for the multi-rate DS-CDMA system with the MPG scheme. In each stage of the adaptive multi-stage PIC receiver, MAI estimates are obtained from the sub-bit estimates from the previous stage and the adaptive weights for the sub-bit estimates. A normalized LMS algorithm is applied to obtain the adaptive weights for the sub-bit estimates. It is shown that the adaptive multi-stage PIC receiver has significant performance improvement over the conventional MF receiver for the multi-rate DS-CDMA system. It is also shown that the adaptive multi-stage PIC receiver achieves smaller BER than the multi-stage PIC receiver and the multi-stage partial PIC receiver for all users with various data rates in a Rayleigh fading channel.
REFERENCES


Fig. 1. Block diagram of s-th stage of the adaptive multi-stage PIC receiver for the multi-rate DS-CDMA system.

Fig. 2. BER of various PIC receivers with 2- and 3-stages for the multi-rate DS-CDMA system with the MPG scheme in a Rayleigh fading channel (for 8 LR, 4 MR, and 2 HR users).