Performance of Matched-Filter Acquisition for a DS/SSMA System in a Frequency-Selective Fading Channel

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Abstract—This paper evaluated the performance of a noncoherent parallel matched-filter acquisition scheme with a reference filter (RF) for a direct-sequence/spread-spectrum multiple access (DS/SSMA) system in a frequency-selective fading channel. Mean acquisition time for the parallel MF-RF acquisition scheme is derived for a frequency-selective Rayleigh fading channel. This acquisition scheme employs reference filter to estimate the variance of interference at the output of detecting matched filter. To apply for multimuser communication environments, multiple access interference is considered in the performance analysis. It is shown that the parallel MF-RF scheme yields smaller mean acquisition time than MF schemes without RF for a frequency-selective Rayleigh fading channel. A parallel MF-RF scheme can be applied to the reverse link (mobile-to-base station) of a CDMA system operating in a frequency-selective fading channel.

I. INTRODUCTION

A direct-sequence/spread-spectrum (DS/SS) system has applications to multiple access (MA) for digital cellular and personal communication systems. For effective and reliable communication in a DS/SSMA system, a local pseudonoise (PN) code at the receiver must be synchronized with the incoming one. The synchronization of a DS/SSMA system consists of two steps: acquisition (coarse alignment) and tracking (fine alignment) [1]. Mainly the former determines the code synchronization time, which should be as short as possible for rapid initial link setup and smooth handoffs in a mobile environment. Because communication cannot take place before acquisition has been achieved, the quick and effective acquisition schemes are required. We focus our attention only on the acquisition problem.

Matched filtering is a commonly used method to acquire the phase of received PN sequence effectively. Performance of a serial MF has been analyzed in an AWGN and a fading channels [2]. To reduce mean acquisition time, a parallel search strategy has been employed [3,4]. A serial MF scheme with reference filter (RF) has been proposed in [5]. A RF is used to estimate the variance of interference at the detecting MF. A parallel MF-RF has been analyzed for a DS/SSMA packet radio system in [4].

Most previous researches have performed on noncoherent I-Q acquisition scheme where PN code synchronization is achieved prior to carrier recovery due to the difficulty in carrier recovery of a wideband low-spectral-density signal. The noncoherent I-Q acquisition scheme can be implemented by fast-decision-rate matched-filter (MF) which enables decision to be made on the order of chip duration or less [6]. In the previous researches, the effect of frequency selectivity on acquisition performance has been rarely considered in a mobile fading channel.

In this paper, the effect of frequency selectivity on acquisition performance is evaluated for a frequency-selective Rayleigh fading channel. The performance measure is mean acquisition time. The mean acquisition time for a parallel MF-RF is derived. The mean acquisition time performance of MF-RF is compared with MF without RF. The frequency-selective fading process is assumed to be wide sense stationary uncorrelated scattering (WSSUS) model which is typical for aircraft/satellite and line-of-sight communications. Because multiple access interference (MAI) is inherent in a DS/SSMA system, the MAI due to multiple users is considered in the analysis [7]. In a DS/SSMA system, MAI is modeled as an AWGN process with variance equal to the MAI variance [8,9]. If we model the MAI components from interfering users as jointly Gaussian random variables, then the MAI components are unconditionally independent.

In Section II, a system model of noncoherent parallel MF-RF is described. In Section III, multiple access interference due to presence of other users is modeled, and the mean acquisition time is derived. Numerical examples are presented in Section IV, and conclusions are drawn in Section V.

II. SYSTEM MODEL

A noncoherent parallel MF acquisition scheme with RF is shown in Fig. 1. It consists of a bank of N noncoherent parallel I-Q MF's with a reference filter. The received signal is first down-converted to inphase and quadrature components. The reference I-Q MF is used to provide a reference level for the synchronization decision. When the output of parallel detecting MF's exceeds the reference MF output by a gain factor, a start signal is sent to the receiver code generator. Then the system goes into a verification mode. The parallel detecting I-Q MF's are loaded with transmitted PN codes and reference I-Q MF is loaded with a PN code orthogonal to the transmitted PN codes. Each of transmitted PN codes consists of $M = T/T_c$, chips where $T$ and $T_c$ are data bit and chip durations, respectively. The
number of taps in a detecting (or reference) I-Q MF is $N/\Delta$ where $\Delta$ is a phase adjustment parameter (or phase updating step size). A running average of the output of a reference I-Q MF is multiplied by a gain factor. The result is used as a decision threshold. A I-Q MF of $N$ parallel detecting MF’s is shown in Fig. 2. A I-Q MF is loaded by one of the $N$ transmitted PN codes.

The acquisition scheme in the analysis has two modes of operation: a search mode and a verification mode (or coincidence detection). In the search mode, a tentative decision is made on the delay of the received signal. In the verification mode, a more accurate decision is achieved to avoid unnecessary false alarms. The following coincidence detection algorithm is employed: acquisition is declared if both A and B test samples exceed a threshold. Asssume that the code uncertainty region is a full code length of $MN$ chips. Each sample at the decision device corresponds to one of the $MN/\Delta$ phases in the code uncertainty region.

III. PERFORMANCE ANALYSIS

In the performance analysis, it is assumed that all samples of MF correlator output are independent, and code uncertainty region is full code length.

A. Frequency-selective Rayleigh Fading Channel

In a typical UHF or microwave land mobile radio channel, Rayleigh fading is encountered for a non-line-of-sight environment. When the multipath spread of the channel is greater than the chip duration of PN code, frequency selectivity should be considered. The WSSUS(wide-sense-stationary-uncorrelated-scattering) fading model is used in the analysis of a variety of digital communication systems. This model is quite general and includes the double selective (i.e. selective in both time and frequency) channels. Only frequency selectivity is considered in this paper.

The channel covariance function for a frequency-selective fading channel is given by

$$g_i(t) = \rho_i(\tau, 0) = \rho_i(\tau, t-s), \text{ for all } t, s. \quad (1)$$

A frequency-selective fading channel exhibits memory and therefore introduces intersymbol interference into the received signal. It is assumed that the selectivity of the channel is such that, in the detection of the data symbol, we need to be concerned only with the two adjacent data symbols. This condition is equivalent to assuuming that $g_i(t) = 0, (|r| > T)$.

Using complex baseband representation for the signals and the impulse response, its autocorrelation function has the form

$$R_{xy}(\tau, \xi, \eta) = E[y(t + \tau, \eta) y^*(t, \xi)] = Q(\tau, \eta) \delta(\xi - \eta). \quad (2)$$

The function $Q(\tau, \eta)$ is called the delay power spectrum (or multipath intensity profile) of the WSSUS channel. In general, the delay power spectrum of a particular radio channel can either be deduced from physical considerations or be measured. Therefore, we assume that $Q(\tau, \eta)$ is a known function. For a Rayleigh fading channel, $Q(\tau, \eta)$ is a stationary zero-mean complex Gaussian random process in time $t$. Because time-discrete pseudonoise sequences are considered here, we derive an appropriate time-discrete channel structure. The following assumptions are necessary: 1) The channel input signal is piecewise constant over intervals of duration $T$, 2) The delay power spectrum is approximately constant for a chip duration, 3) The channel distortion in time is limited to M intervals of length $T$, 4) The channel is approximately constant for PN code period. If the assumption 4) is valid, all signals transmitted within one data symbol interval experience essentially the same channel impulse response. The assumption 4) is roughly equivalent to the statement that the channel's coherence time is greater than the data symbol length. It the above assumptions hold, the channel can be modeled such that the output signal prior to the AWGN is constant during a chip interval.

B. MAI Modeling

In the reverse links of a CDMA system, other users transmit with orthogonal PN codes and their signals fade independently. The MAI contribution at the output of MF correlator is basically a Gaussian.

The $k$th transmitted signal for a DS/SSMA system is given by

$$s_k(t) = \sqrt{2P_k} d_k(t) c_k(t) \cos(\omega_c t + \phi_k), \quad (3)$$

where $P_k$ is the signal power, $d_k(t)$ is data sequence, $c_k(t)$ is PN code sequence, $\omega_c$ is phase parameter of carrier. Assume that $P_k$ is independent of the propagation delay and phase parameter. Suppose $d_k(t)$ and $c_k(t)$ are sequences of $-1$ with rectangular pulse shape. The spreading sequence $c_k(t)$ is generated at a rate $M$ chips per data pulse. During demodulation at the receiver, the composite signal is multiplied by a replica of the spreading sequence so that the desired data sequence is obtained at the receiver output.

The received signal by the $k$th receiver is given by

$$r_k(t) = D(t) + I(t), \quad (4)$$

where $D(t)$ is desired signal term and $I(t)$ is interference signal term. The received specular multipath signal transmitted by the $k$th transmitter is given by

$$R_k(t) = \sum_{l=1}^{L_k} x_l(t - \tau_{k,l}), \quad (5)$$

where the $l$th received specular multipath signal transmitted by the $k$th transmitter is

$$x_l(t - \tau_{k,l}) = \alpha_{k,l} d_k(t - \tau_{k,l}) c_k(t - \tau_{k,l}) \cos(\omega_c t + \phi_{k,l}), \quad (6)$$

$L_k$ is the number of multipath components, and $\tau_{k,1} < \tau_{k,2} < \cdots < \tau_{k,L_k}$.

Interference components $I(t)$ consists of AWGN and MAI of intracell and intercell, and is given by
where the first term is the MAI within the cell, the second term is the MAI outside the cell, the third term is AWGN with noise power spectral density $N_0/2$, $K_1$ is the number of users within the cell, and $K_2$ is the number of users outside the cell. Let $Z_{k,i}$ denote the decision variable for the $l$th specular multipath signal transmitted by the $k$th transmitter. The contribution of the $k'$th specular multipath signal of the $Z$th transmitter on $Z_{k,l}$ is given by

$$Z_{k,l}' = \frac{\alpha_{k,l}^{t}}{\sqrt{2T}} \cos(\phi_{k'},\nu - \phi_{k_l}) \int_{\tau_{k_l}}^{T} d_{k}(t - \tau_{k,l})c_{k}(t - \tau_{k,l})dt$$

$$= \frac{\alpha_{k,l}^{t}}{\sqrt{2T}} h_{k,l}'$$

Note that $\gamma_{k,l}'$ are uncorrelated for $k \neq k'$. $\gamma_{k,l}'$ can be approximated by a Gaussian random variable with zero mean and average variance which is averaged over the delay $\tau$ and phase $\phi$. Then the average variance of $\gamma_{k,l}'$ is given by

$$\sigma_{\gamma}^2 = \frac{MT_c^2}{3},$$

where $T_c$ is chip duration, $M$ is processing gain given by $M = T/T_c$, and $T$ is bit duration. It is assumed that the reference user(desired user) user is user 0. Then $\gamma_{k,l}$ has the following Gaussian distribution with mean $d_0$ and variance

$$\frac{1}{3M} \left[ \sum_{l=1}^{L_0} \sum_{i,m=1}^{L_1} \alpha_{k,l}^{t} \gamma_{l,i}^{t} + K_1 \sum_{l=1}^{L_0} \sum_{i,m=1}^{L_1} \alpha_{k,l}^{t} \gamma_{l,i}^{t} + \sum_{j=1}^{K_2} \sum_{l=1}^{L_0} \sum_{i,m=1}^{L_1} \alpha_{k,l}^{t} \gamma_{l,i}^{t} - \frac{N_0}{\alpha_{k,l}^{t} T} \right].$$

Then the total signal-to-interference ratio (SIR) is given by

$$\frac{E_{k,l}^{b,l}}{N_0 + \frac{1}{3M} \left[ \sum_{l=1}^{L_0} E_{k,l}^{b,m} + \sum_{i=1}^{L_1} \sum_{m=1}^{L_1} E_{k,l}^{i,m} + \sum_{j=1}^{K_2} \sum_{l=1}^{L_0} E_{k,l}^{t} \right]},$$

where $E_{k,l}^{b,l} = \alpha_{k,l}^{t} I^{b,l}/T/2$ is bit energy of the $l$th specular multipath signal of the $k$th transmitter.

### C. Acquisition Performance

With a bandpass filter (BPF) of a bandwidth $B = 2/T_c$ at the receiver frontend, it was shown that $M/N/\Delta$ samples from $N$ noncoherent parallel MF-RF are mutually independent and average noise in BPF output is $\sigma^2 = N_0/T_c$ [1]. In the acquisition performance analysis, the total SIR is incorporated into the derivations of detection and false alarm probabilities instead of only AWGN. The acquisition process can be modelled using state transition diagram shown in Fig. 3. The $H_1$ is the hypothesis where acquisition occurs and the $H_0$ is the hypothesis where false alarm occurs. In a frequency selective fading channel, the $H_1$ region consists of more than one cell due to multipath delay spread. It is assumed that the code tracking loop can track PN code if initial phase offset between the local PN code and the transmitted PN code is at most 0.5 chip. When the initial phase offset in the $H_1$ is equal to 0.5 $T_c$, the number of $H_1$ subcells is $2[D_0/T_c] + 2$ in a frequency selective fading case where $[x]$ is the integer part of $x$. $D_0$ is delay spread.

In Fig. 3, the transfer parameters of each state are given as follows:

$$H_D(Z) = P_{D1,i} P_{D2,i} + \sum_{i=2}^{e} [P_{D1,i} P_{D2,i} Z^{(2AM+1)d}]$$

$$= \prod_{j=1}^{e} (1 - F_{D1,j}) Z^d + P_{D1,i} (1 - F_{D2,i}) Z^{(2AM+1)d},$$

$$H_M(Z) = \prod_{j=1}^{e} (1 - F_{D1,j}) Z^d + P_{D1,i} (1 - F_{D2,i}) Z^{(2AM+1)d},$$

$$H_{FA}(Z) = P_{F1} P_{F2} Z^{(2AM+1)d},$$

$$P_{DF1} = \sum_{j=0}^{A} \left( A \right) P_{D1,i} (1 - P_{D1,i})^{(A-j)},$$

$$H_{NF}(Z) = (1 - P_{F1}) Z^d + P_{F1} (1 - P_{F2}) Z^{(2AM+1)d},$$

$$P_{DF2} = \sum_{j=0}^{A} \left( A \right) P_{F1}^{j} (1 - P_{F1})^{(A-j)},$$

where $P_{DF1,i}$ is detection probability in search mode at subcell $i$ of $H_1$, $P_{DF2,i}$ is detection probability in verification mode at subcell $i$ of $H_1$, $P_{DF1}$ is false alarm probability in search mode, $P_{DF2}$ is false alarm probability in verification mode, $H_{M}(Z)$ is miss detection probability, $x$ is the number of $H_2$ subcells.

When the penalty time due to false alarm is $Jd$, $H_{P}(Z)$ is given by

$$H_{P}(Z) = Z^{ld}. $$

Using straight-line approach for uniformly-distributed starting code phase offset over code uncertainty region, mean acquisition time is obtained by

$$E[T_{acq}] = \left[ \frac{1}{H_D(Z)_{Z=1}} \right] \cdot \left[ \frac{d}{dZ} H_D(Z)_{Z=1} \right] + \left[ \frac{1}{H_M(Z)_{Z=1}} \right] \cdot \left[ \frac{d}{dZ} H_M(Z)_{Z=1} \right] + \left[ (\nu - 1) \frac{d}{dZ} H_{NF}(Z)_{Z=1} + H_{FA}(Z) H_{P}(Z)_{Z=1} \right] \cdot \left[ \frac{1 - H_D(Z)_{Z=1}}{2} \right],$$

where $\nu - 1$ is the number of cells of $H_0$. 

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IV. NUMERICAL RESULTS

We compute the mean acquisition time of a noncoherent parallel MF-RF for a frequency-selective Rayleigh fading channel. The thresholds for the search and verification modes are selected numerically to minimize mean acquisition time for each SNR/chip. For numerical examples, the following parameters are used: 1) penalty time due to a false alarm in the verification mode is $10^6$ (chips), 2) $A = 4$ and $B = 2$, 3) phase adjustment parameter $\Delta = 1/2$, 4) code length $= 212 - 1$. The gain factor of RF is chosen to ensure constant false alarm probability $P_{fa} \leq 10^{-6}$.

In Fig. 4, the normalized mean acquisition time vs. SNR/chip is compared for serial and parallel MF and MF-RFs in a frequency-selective Rayleigh fading channel. The numerical examples are shown for a normalized Doppler frequency $f_dT_c = 10^{-4}$, the number of users within the cell $K_1 = 10$, the number of users outside the cell $K_2 = 100$, delay spread $D_s = 3T_c$, the number of detecting parallel MFs $N = 10$, and MF length $M = 128$. The mean acquisition time is normalized by chip duration $T_c$. It is shown that the parallel acquisition scheme achieves faster acquisition than serial acquisition scheme regardless of the presence of RF. The parallel MF-RF scheme yields the smallest mean acquisition time among four schemes. Also, it can be noted that reference filter gives the advantage in MA applications because it is used to estimate the variance of MAI at the output of the detecting MF.

In Fig. 5, the normalized mean acquisition time vs. SNR/chip is shown for a parallel MF-RF in a frequency-selective Rayleigh fading channel with delay spread as a parameter. The numerical examples are shown for $f_dT_c = 10^{-4}$, $K_1 = 10$, $K_2 = 100$, $N = 10$, and $M = 128$. It is shown that large delay spread increases mean acquisition time.

In Fig. 6, the normalized mean acquisition time vs. SNR/chip is shown for a parallel MF-RF in a frequency-selective Rayleigh fading channel with the number of users within the cell as a parameter. The numerical examples are shown for $f_dT_c = 10^{-4}$, $D_s = 3T_c$, $K_2 = 100$, $N = 10$, and $M = 128$. It is shown that as expected, the larger number of users within the cell increases mean acquisition time. That is, the MAI effect on acquisition performance is more significant for the users within the cell than the users outside the cell.

In Fig. 7, the normalized mean acquisition time vs. SNR/chip is shown for a parallel MF-RF in a frequency-selective Rayleigh fading channel with the number of users outside the cell as a parameter. The numerical examples are shown for $f_dT_c = 10^{-4}$, $D_s = 3T_c$, $K_1 = 50$, $N = 10$, and $M = 128$. It is shown that as expected, the larger number of users outside the cell increases mean acquisition time. The MAI from users outside the cell has more significant influence on acquisition performance than the users outside the cell. From Fig. 6 and Fig. 7, it is generally expected that the mean acquisition time may not be significantly increased due to the users outside cell because the propagation loss is severe from the users outside the cell.

REFERENCES

Fig. 1. Noncoherent parallel MF-RF acquisition scheme.

Fig. 2. Noncoherent I-Q MF acquisition scheme.

Fig. 3. State transition diagram for acquisition process.

Fig. 5. Normalized mean acquisition time vs. SNR/chip for a parallel MF-RF with delay spread as a parameter.

Fig. 6. Normalized mean acquisition time vs. SNR/chip for a parallel MF-RF with the number of users within the cell as a parameter.

Fig. 4. Normalized mean acquisition time vs. SNR/chip for serial MF and MF-RF, and parallel MF and MF-RF.

Fig. 7. Normalized mean acquisition time vs. SNR/chip for a parallel MF-RF with the number of users outside the cell as a parameter.