Efficient Subcarrier and Bit Allocation Algorithm for OFDMA System with Adaptive Modulation

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Abstract—This paper investigates the adaptive subcarrier and bit allocation algorithm for OFDMA systems. To minimize overall transmit power, we propose a novel adaptive subcarrier and bit allocation algorithm based on channel state information. Moreover, the block-wise method is considered based on channel variation for adaptive subcarrier and bit allocation. It is shown that a near optimal solution is obtained by the proposed algorithm which has low complexity compared to that of other conventional algorithms. Also, it is shown that the block-wise method significantly reduces the complexity and the feedback or side information though slight transmit power increase.

Keywords - adaptive subcarrier allocation, transmit power, complexity

I. INTRODUCTION

Recently, orthogonal frequency division multiple access (OFDMA) has emerged as a major multiple access scheme for new wireless communication systems [1], [2]. High spectral efficiency can be obtained by OFDMA in which inter-symbol interference (ISI) is eliminated over multi-path channels. Moreover, significant performance improvement can be achieved by using adaptive modulation in combination with OFDMA over frequency selective fading channels [3].

In a single-user OFDM system, an adaptive bit allocation (ABA) algorithm, namely the greedy algorithm, gives an optimal solution to minimize overall transmit power for adaptive modulation [4]. In the OFDMA system, however, the greedy algorithm cannot give an optimal solution since the best subcarrier for a user can be the best subcarrier also for another user. An optimal solution can be obtained by the greedy algorithm when overall subcarriers are allocated to users by adaptive subcarrier allocation (ASA) in the OFDMA system [5].

In this paper, the optimization problem to minimize overall transmit power is formulated. Based on the formulation, we propose the novel ASA algorithm and the complexity reduction method is also presented.

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II. SYSTEM MODEL

Consider a downlink OFDMA system with $K$ users and $N$ subcarriers. $R_k$ bits per OFDM symbol are transmitted for the $k$-th user. Assume that each subcarrier has a bandwidth that is much smaller than the coherence bandwidth of the channel, so total bandwidth is divided into many flat subchannels. In the frequency selective fading channel, different subcarriers may have different single complex channel gains. Assume that the magnitude of channel gain $g_{k,n}$ for the $k$-th user on the $n$-th subcarrier is perfectly known to both the transmitter and the receiver in coherent reception. Suppose that $f_k(c)$ is the required received power of the $k$-th user to achieve the desired BER in the reception of $c$ bits. The subcarrier allocation indicator is defined as

$$P_{k,n} = \begin{cases} 1, & \text{if } n\text{-th subcarrier is allocated to } k\text{-th user,} \\ 0, & \text{otherwise.} \end{cases}$$

The objective of the ASA and ABA algorithm is to find the best subcarrier and bit allocation minimizing overall transmit power for given transmission rates and BER requirements of users. The transmit power for the $k$-th user on the $n$-th subcarrier is given by

$$P_{k,n} = \frac{f_k(c_{k,n})}{g_{k,n}^2},$$

where $c_{k,n}$ is the number of bits allocated to the $n$-th subcarrier for the $k$-th user. Thus the optimization problem to minimize overall transmit power is formulated as

$$\min_{P_k} \sum_{k=1}^{K} \sum_{n=1}^{N} P_{k,n} = \min_{c_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} P_{k,n} f_k(c_{k,n})$$

subject to the following constraints:

$$R_k = \sum_{n=1}^{N} c_{k,n} \text{ for all } k \in \{1, 2, ..., K\},$$

$$\sum_{k=1}^{K} R_k \leq ND_{\text{max}} \text{ for all } k \in \{1, 2, ..., K\},$$
\[
\sum_{\alpha=1}^{\kappa} \rho_{\alpha,n} = 1 \text{ for all } n \in \{1,2,...,N\},
\]

where \( D \) is a set of the possible non-negative integer numbers of bits on a subcarrier and \( D_{\text{max}} \) is the maximum number of bits per subcarrier.

The above optimization problem is the combinatorial optimization problem with respect to \( c_{k,n} \) and \( \rho_{\alpha,n} \), which requires many computations to solve. For only ASA, we need to know not the set of the numbers of bits \( \{c_{k,n}\} \) but only the set of the subcarrier allocation indicators \( \{\rho_{\alpha,n}\} \). Thus, instead of using \( c_{k,n} \) and \( g_{l,m} \), we modify the problem formulation (3) by using the average number of bits per subcarrier and the average channel gain, respectively, on allocated subcarriers for each user. Let \( s_k \) be the number of allocated subcarriers for the \( k \)-th user, and \( G_i \) be the average channel gain square of allocated subcarriers for the \( k \)-th user, which are given by

\[
G_i = \frac{\sum_{\alpha=1}^{N} \rho_{\alpha,0} g_{i,\alpha}^2}{s_k},
\]

respectively, then the above optimization problem is reformulated as

\[
\min P_i = \min \left\{ \frac{s_k}{G_i} \sum_{\alpha=1}^{K} \frac{\rho_{\alpha,0} g_{\alpha}^2}{s_k} \right\} = \min \left\{ \frac{\sum_{\alpha=1}^{N} \rho_{\alpha,0} g_{\alpha}^2}{s_k} \right\}
\]

subject to the following constraints:

\[
\left[ \frac{R_f}{D_{\text{max}}} \right] \leq \sum_{\alpha=1}^{N} \rho_{\alpha,0} \leq N \text{ for all } k \in \{1,2,...,K\},
\]

\[
\sum_{k=1}^{K} R_k \leq ND_{\text{max}} \text{ for all } k \in \{1,2,...,K\},
\]

\[
\sum_{\alpha=1}^{N} \rho_{\alpha,n} = 1 \text{ for all } n \in \{1,2,...,N\},
\]

where \( \left[ \frac{R_f}{D_{\text{max}}} \right] \) is the minimum required number of subcarriers for the \( k \)-th user.

In the above problem formulation, we use the average number of bits and the average channel gain based on the information of the actually allocated subcarriers. To use the information of the actually allocated subcarriers increases the system performance. The performance increase will be shown in numerical results. The formulation (7) contains only \( \{\rho_{\alpha,n}\} \) for variables and it can be easily solved by the simple greedy method.

### III. EFFICIENT ASA AND ABA APPROACHES

#### A. Novel ASA algorithm

Based on (7) and (8), we propose a novel ASA algorithm to minimize overall transmit power. The proposed ASA algorithm consists of the initial subcarrier allocation and the residual subcarrier allocation. Assume that the \( n \)-th subcarrier is selected for \( n \in \{1,2,3,...,N\} \). Then, the average channel gain on the selected subcarrier and pre-allocated subcarriers is given by

\[
G_i = \left( \frac{\sum_{\alpha=1}^{N} \rho_{\alpha,n} g_{i,\alpha}^2}{s_k} \right) (s_k + 1).
\]

Before subcarriers are allocated in the initial subcarrier allocation, the fixed average number of bits \( c_i^* \) is firstly determined since the variable average number of bits, \( R_i/s_k \), is too large so that the function \( f_i(\cdot) \) is not feasible. The fixed average number of bits \( c_i^* \) is obtained by

\[
c_i^* = \frac{R_i}{s_k}
\]

where \( s_k^* \) is the fixed number of subcarriers which is obtained by the 'greedy approach' in [6].

For the initial subcarrier allocation, a subcarrier is allocated to the \( k \)-th user, which is obtained as

\[
k = \arg \min_{k \in \{1,2,...,K\}} \Delta P_i,
\]

where \( \Delta P_i \) is the transmit power decrease which is given by

\[
\Delta P_i = \frac{1}{G_i} f_i \left( \frac{R_i}{s_k^*} \right).
\]

Then, update the subcarrier allocation indicator, the number of subcarriers, and the average channel gain as \( \rho_{k,0} = 1 \), \( s_k^* = s_k + 1 \), and \( G_i = G_i^* \), respectively. The subcarriers are allocated to users for the initial subcarrier allocation until the number of subcarriers \( s_k \) becomes more than \( \left[ \frac{R_i}{D_{\text{max}}} \right] \).

For the residual subcarrier allocation, a subcarrier is allocated to the \( k \)-th user, which is obtained as

\[
k = \arg \min_{k \in \{1,2,...,K\}} \Delta P_i,
\]

where \( \Delta P_i \) is the transmit power decrease which is given by

\[
\Delta P_i = \frac{1}{G_i} f_i \left( \frac{R_i}{s_k^*} \right).
\]

Then, update the subcarrier allocation indicator, the number of subcarriers, and the average channel gain as \( \rho_{k,0} = 1 \), \( s_k^* = s_k + 1 \), and \( G_i = G_i^* \), respectively. The subcarriers are allocated to users for the residual subcarrier allocation until overall subcarriers are allocated. For the ABA, the data bits are allocated to each user by using the greedy algorithm in [4].
as single-user OFDM systems. The details of the proposed algorithm is described as follows:

**Initial subcarrier allocation**

1. Calculate $C_i$ for all $i \in \{1,2,...,K\}$.
2. Re-order subcarriers randomly, set $N=\{1,2,...,N\}$, $K=\{1,2,...,K\}$, $i=K$.
3. Allocate $K$ subcarriers with maximum channel gains for users.
4. Find $k = \arg\min_{i} \Delta P_i$, and update $\rho_{kn}, s_i, G_i$.
5. Repeat the Step 3 and 4, until $s_i = \left[ R_i / D_{max} \right]$ for $k \in \{1,2,...,K\}$. Then, go to the residual subcarrier allocation.

**Residual subcarrier allocation**

1. Find $\hat{k} = \arg\min_{i} \Delta P_i$, and update $\rho_{kn}, s_i, G_i$.
2. Repeat the Step 1, until overall subcarriers are allocated.

When one of the subcarriers is selected in the initial subcarrier allocation or the residual subcarrier allocation, $O(K)$ calculations are required. Thus, total $O(N \times K)$ calculations are required during the initial allocation and the constructive allocation part. In Table. 1, the complexity of the proposed ASA algorithm is compared with the conventional ASA algorithms in [5], [6], and [7], namely MAO, ACG, and Vogel algorithm, respectively, where the Vogel algorithm in [7] is a solution of the package problems which are mentioned in [8]. The proposed ASA algorithm has the complexity similar to the ACG algorithm, and much lower than the complexity of other ASA algorithms.

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**B. Block-wise method for ASA and ABA algorithms**

Multi-path in the wireless channel creates rapid changes in signal strength over a small travel distance or time interval, which cause fluctuations in signal strength in the frequency domain. Thus, the subcarriers may have different channel gains due to different paths and different delay spreads in the frequency selective fading channel. However, signals on the adjacent subcarriers are still strongly correlated in amplitude. Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered to have approximately equal gain and linear phase. The coherence bandwidth $B_c$ is a defined relation derived from the RMS(root mean square) delay spread $\tau_m$.

If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function (FCF) is above 0.5, then the coherence bandwidth is approximately

$$B_c = \frac{1}{5\tau_m^2},$$

where $\tau_m$ denotes the root mean square of delay spread [14]. When the bandwidth of adjacent subcarriers is narrower than the coherence bandwidth, the channel gains of the adjacent subcarriers may be regarded as similar to each other. With the above motivations, we propose the block-wise method for the ASA and ABA, where 'block' means a group of adjacent subcarriers.

Let a block size $b$ be the number of adjacent subcarriers in the group, then the block size $b$ can be determined as

$$b \leq \frac{B_c}{\Delta f} = \frac{N}{5\tau_m B},$$

where $\Delta f$ denotes the subchannel bandwidth occupied by a subcarrier and $B$ denotes the total channel bandwidth for the system. Also, the block size $b$ can be determined from the required system performance and complexity.

In the proposed method, it is given the same value to the channel gains of adjacent subcarriers in the same block, which value is defined as a 'block channel gain'. The block channel gain of the $m$-th block for the $k$-th user is defined as

$$g_{km} = \min_n g_{kn}$$

where $M$ is the number of blocks which is given by

$$M = \left\lceil \frac{N}{b} \right\rceil.$$

Then it can be applied to ASA and ABA using the parameter $M$ and $g_{km}$ instead of $N$ and $g_{kn}$, respectively. And the subcarriers in a block have to be allocated to all users and the same number of bits are allocated to the subcarriers in a block even as single subcarrier.

By using the proposed block-wise method, the complexity of ASA and ABA algorithms can be reduced as $b$ times, because the number of blocks $M$ is used instead of the number of subcarriers $N$, which is a key factor to determine the complexity of ASA and ABA algorithm. Furthermore, the required amount of the side information is also reduced as $b$ times, so that it prevents dissipation of the limited frequency bandwidth and transmit power from increasing.

**IV. Simulation Results**

Consider an OFDMA system with 2,048 subcarriers over 20 MHz bandwidth. Total of 8,192 bits is transmitted in each
OFDM symbol making spectral efficiency about 4 bits/sec/Hz. Assume that each user undergoes the 5-path frequency selective Rayleigh fading channel with an exponential delay profile. For adaptive modulation, QPSK, 16-QAM, and 64-QAM are employed. In this section, performance of the proposed ASA algorithm is compared with that of the conventional ASA algorithms in [5], [6], and [7] in addition to a fixed subcarrier allocation (FSA) which allocates predetermined subcarriers to users. To ensure a fair comparison, the greedy algorithm for the ABA is used to all the ASA and FSA algorithms.

Fig. 3 shows the average BER of the OFDMA systems by using FSA and ASA algorithms for $K=16$ and $\tau_{\text{rms}}=5\mu s$. It is shown that the ASA achieves much more gain than the FSA.

OFDMA systems for $K=16$ and $\tau_{\text{rms}}=5\mu s$.

For the ASA, the proposed algorithm achieves more gains of 1.3 dB and 4.8 dB than the Vogel algorithm and the ACG algorithm, respectively, which is a near optimal performance as that of MAO algorithm. Fig. 4 shows the required $E_s/N_0$ to achieve average BER of $10^{-5}$ of the OFDMA systems by using FSA and ASA algorithms for $\tau_{\text{rms}}=5\mu s$. The required $E_s/N_0$ of the FSA is not variant as the number of users increases, while the required $E_s/N_0$ of the ASA is reduced. Such a multiuser diversity can be obtained by using ASA because the probability that each frequency has better channel gains is increased by increasing the number of users. In Fig. 3, it is shown that the proposed algorithm achieves much more gain than the conventional algorithms.

Fig. 4. Average bit error rate versus average $E_s/N_0$ in various OFDMA systems for $K=16$ and $\tau_{\text{rms}}=5\mu s$. 

Fig. 5. Average bit error rate versus average $E_s/N_0$ in the OFDMA system by using block-wise ASA and ABA for $K=16$ and $\tau_{\text{rms}}=500\mu s$.

Fig. 6. Required $E_s/N_0$ to achieve average BER of $10^{-5}$ according to RMS delay spread of frequency selective channels for $K=16$ and various block sizes.

Fig. 3. Average bit error rate versus average $E_s/N_0$ in various OFDMA systems for $K=16$ and $\tau_{\text{rms}}=5\mu s$.

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Fig. 6. Required $E_s/N_0$ to achieve average BER of $10^{-5}$ according to RMS delay spread of frequency selective channels for $K=16$ and various block sizes.
In Fig. 5, it is shown that the average BER of the OFDMA system by using the proposed ASA algorithm based on the block-wise method for $K=16$ and $\tau_{\text{ms}}=500\text{ns}$. Frequency selective fading channel with $\tau_{\text{ms}}=500\text{ns}$ may have a coherence bandwidth of $B_c=400\text{ kHz}$ when the frequency correlation function is above 0.5. The channel bandwidth occupied by a subcarrier is $9.765\text{ kHz}$, so that the block size $b$ can be determined as less than 40. When the block size is determined as 32, as shown in Fig. 5, the average $E_b/N_0$ increases by at most 1.8 dB at average BER of $10^{-3}$, the complexity of the ASA algorithm as well as the required amount of the side information are reduced as 32 times. Fig. 6 shows that the required $E_b/N_0$ to achieve average BER of $10^{-3}$ of the OFDMA system by using the proposed ASA algorithm based on the block-wise method for $K=16$ and various block sizes. As shown in Fig. 6, the block size to maintain the required performance is decreased as RMS delay spread increases. If we determine the block size below the coherence bandwidth of $B_c=50\text{ kHz}$, the complexity and the amount of the side information are reduced as $b$ times, while less than at most 3 dB transmit power increases.

V. CONCLUSIONS

In this paper, we investigate an ASA algorithm to minimize overall transmit power. For solving the complex optimization problem easily, the problem to minimize overall transmit power is formulated by using the average number of bits per subcarrier and the average channel gain based on the actually allocated subcarriers. From the formulation, we propose a novel ASA algorithm which has low complexity. Also, we present the block-wise method to reduce the complexity and the amount of side information for the ASA and ABA algorithm. It is shown that the proposed algorithm gives a near optimal solution to minimize overall transmit power and has low complexity compared with conventional ASA algorithms. Also, it is shown that the complexity as well as the amount of the side information are significantly reduced by using the block-wise method, while transmit power increase is not significant compared with that of the non-block-wise method.

VI. REFERENCES