

## Multiple Trellis Coded Differential Unitary Space-Time Modulation

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**Abstract**— In this paper, differential unitary space-time modulation (DUSTM) is combined with multiple trellis codes. The set partitioning of a unitary space-time constellation for multiple trellis code is described, and several codes constructed are provided. The computational complexity of multiple trellis coded DUSTM is considered. Numerical results are given.

### I. INTRODUCTION

Differential unitary space-time modulation is a scheme proposed for the transmit antenna diversity to combat detrimental effects in wireless fading channels at a receiver without channel state information [1], [2]. In the DUSTM, the transmitted signal matrix at each time block is the product of the previously transmitted signal matrix and the current unitary data matrix. The constellations for unitary space-time modulated signals form groups under matrix multiplication.

Since the publications of [1], [2], several works to improve performance have been introduced. These works are divided into two categories. One is to find constellations having better properties [3], [4]. In [3], the optimal unitary cyclic group constellations are found. In [4], the optimal unitary non-group constellations are found. The other is to combine the DUSTM with conventional channel coding [5], [6]. In [5], the combination of the DUSTM and turbo codes is provided. In [6], the combination of the DUSTM and trellis codes is provided.

Multiple trellis coded modulation (MTCM), proposed by Divsalar *et al.* [7], provides additional coding gain by maximizing the minimum product of Euclidean distances with the minimum Hamming distance for fading channels. In this paper, the DUSTM is combined with multiple trellis codes. The set partitioning of a unitary space-time constellation for multiple trellis code is described, and several codes constructed are provided. The computational complexity of multiple trellis coded (MTC) DUSTM is compared with that of trellis coded DUSTM. Numerical results are given, and conclusions are drawn.

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### II. CHANNEL MODEL AND DUSTM

#### A. Channel Model

Consider a wireless channel in which data are sent from  $n_T$  transmit antennas to  $n_R$  receive antennas. It is assumed that the channel has flat Rayleigh fading and the channel coefficients for different transmit-receive antenna pairs are statistically independent and remain unchanged during  $T$  time intervals. Let  $c_t^j$  denote the transmitted signal by the transmit antenna  $j$  at the time  $t$  where  $j=1, \dots, n_T$  and  $t=1, \dots, T$ . The received signal  $y_t^i$  for the receive antenna  $i$  at the time  $t$  is given by

$$y_t^i = \sqrt{\rho} \sum_{j=1}^{n_T} h_{ji} c_t^j + n_t^i, \quad i=1, 2, \dots, n_R, \quad (1)$$

where  $\rho$  is the signal-to-noise ratio (SNR) per receive antenna,  $h_{ji}$  is the zero-mean complex Gaussian channel coefficient from the transmit antenna  $j$  to the receive antenna  $i$  with variance 0.5 per dimension, and  $n_t^i$  is the zero-mean complex additive white Gaussian noise (AWGN) with variance 0.5 per dimension. Assume that the power of transmitted signals is normalized to unit power. The received signal in matrix form, received signal matrix, is written as

$$\mathbf{Y} = \sqrt{\rho} \mathbf{C} \mathbf{H} + \mathbf{N} \quad (2)$$

where  $\mathbf{C} = \{c_t^j\}$  is the  $T \times n_T$  transmitted signal matrix,  $\mathbf{H} = \{h_{ji}\}$  is the  $n_T \times n_R$  channel matrix with independent and identically distributed (i.i.d.) entries, and  $\mathbf{N} = \{n_t^i\}$  is the  $T \times n_R$  noise matrix with i.i.d. entries. For an unknown channel matrix  $\mathbf{H}$ , the conditional probability density function (pdf) of the received signal matrix  $\mathbf{Y}$  given a transmitted signal matrix  $\mathbf{C}$  is given by [1]

$$p(\mathbf{Y} | \mathbf{C}) = \frac{\exp\left(-\text{tr}\left\{\mathbf{Y}^\dagger (\mathbf{I} + \rho \mathbf{C} \mathbf{C}^\dagger)^{-1} \mathbf{Y}\right\}\right)}{\pi^{n_R T} \det(\mathbf{I} + \rho \mathbf{C} \mathbf{C}^\dagger)^{n_R}} \quad (3)$$

where “tr” and “ $\dagger$ ” stand for trace and conjugate transpose, respectively.

#### B. Differential Unitary Space-Time Modulation (DUSTM)

Let  $\mathbf{V}_k$  denote the  $n_T \times n_T$  data matrix at the  $k$ th block where only DUSTM with  $T = n_T$  is considered. The data matrix  $\mathbf{V}_k$  forms a unitary space-time constellation  $\mathcal{V}$  with

cardinality  $L$ . i.e.,  $\mathcal{V} \equiv \{\mathbf{V}_l \mid \mathbf{V}_l \mathbf{V}_l^T = \mathbf{I}, l = 0, \dots, L-1\}$  where  $\mathbf{I}$  is the identity matrix. Let  $\mathbf{C}_k$  denote the  $n_t \times n_t$  transmitted signal matrix at the  $k$ th block. This matrix is given by

$$\mathbf{C}_k = \mathbf{V}_k \mathbf{C}_{k-1}, \mathbf{C}_0 = \mathbf{I}. \quad (4)$$

The received signal matrix for  $\mathbf{C}_k$  is given by

$$\mathbf{Y}_k = \sqrt{\rho} \mathbf{C}_k \mathbf{H} + \mathbf{N}_k \quad (5)$$

where  $\mathbf{N}_k$  is the noise matrix at the  $k$ th block, and  $\mathbf{H}$  is the channel matrix which is constant during two consecutive blocks. As DPSK, a DUSTM receiver estimates data matrix  $\mathbf{V}_k$  by observing two consecutive received signal matrices

$$\bar{\mathbf{Y}}_k \triangleq [\mathbf{Y}_{k-1}^T : \mathbf{Y}_k^T]^T$$

where “ $T$ ” stands for transpose. Two transmitted signal matrices that affect  $\bar{\mathbf{Y}}_k$  are  $\bar{\mathbf{C}}_k = [\mathbf{C}_{k-1}^T : \mathbf{C}_k^T]^T$ . From (3), the conditional pdf of  $\bar{\mathbf{Y}}_k$  given  $\bar{\mathbf{C}}_k$  is given by [1]

$$p(\bar{\mathbf{Y}}_k \mid \bar{\mathbf{C}}_k) = \frac{\exp\left(-\text{tr}\left\{\bar{\mathbf{Y}}_k^i \left[\mathbf{I} - \frac{\rho}{1+2\rho} \begin{bmatrix} \mathbf{I} \\ \vdots \\ \mathbf{V}_k^i \end{bmatrix} [\mathbf{I} : \mathbf{V}_k^i] \bar{\mathbf{Y}}_k\right]\right\}\right)}{\pi^{2n_t n_k} (1+2\rho)^{n_t n_k}} \\ = p(\bar{\mathbf{Y}}_k \mid \mathbf{V}_k). \quad (6)$$

The differential demodulation, which maximizes the conditional pdf of (6), is given by

$$\hat{\mathbf{V}}_k = \arg \max_{\mathbf{V} \in \mathcal{V}} \text{tr}\left\{\bar{\mathbf{Y}}_k^i \begin{bmatrix} \mathbf{I} \\ \vdots \\ \mathbf{V}^i \end{bmatrix} [\mathbf{I} : \mathbf{V}^i] \bar{\mathbf{Y}}_k\right\} \\ = \arg \max_{\mathbf{V} \in \mathcal{V}} \text{Re tr}\{\mathbf{Y}_{k-1} \mathbf{Y}_k^i \mathbf{V}\}. \quad (7)$$

### C. Trellis Coded DUSTM

Fig. 1 shows the transmitter in trellis coded DUSTM. Information bits are encoded by a rate  $b/n$  trellis code. The encoded bit stream is divided into the blocks of  $n$  bits each of which is mapped into an element in the unitary space-time constellation  $\mathcal{V}$ . After matrix-wise interleaving, the unitary space-time mapped signals are differentially encoded and transmitted by  $n_t$  transmit antennas over a slow flat Rayleigh fading channel.

Let  $\mathbf{Y}$  and  $\mathbf{V}$  denote the received signal matrix sequence  $\{\mathbf{Y}_0, \mathbf{Y}_1, \dots\}$  and the coded signal matrix sequence  $\{\mathbf{V}_0, \mathbf{V}_1, \dots\}$ , respectively. The joint conditional pdf of  $\mathbf{Y}$  given  $\mathbf{V}$  is given by [6]

$$p(\mathbf{Y} \mid \mathbf{V}) = \prod_k p(\bar{\mathbf{Y}}_k \mid \bar{\mathbf{V}}_k) = \prod_k p(\bar{\mathbf{Y}}_k \mid \bar{\mathbf{C}}_k). \quad (8)$$

The pairwise error probability, i.e., the probability of incorrectly decoding  $\mathbf{V}$  to  $\mathbf{U}$ , is given by [6]

$$p(\mathbf{V} \rightarrow \mathbf{U}) \leq \prod_k \det\left(\mathbf{I} + \frac{\rho^2}{4(1+2\rho)} (\mathbf{V}_k - \mathbf{U}_k)^i (\mathbf{V}_k - \mathbf{U}_k)\right)^{-n_k} \quad (9)$$

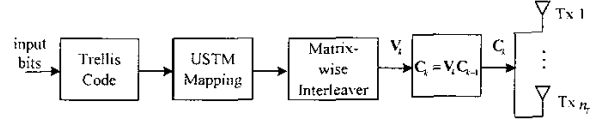


Fig. 1. Transmitter in trellis coded DUSTM

Let  $\eta(\mathbf{V}, \mathbf{U})$  denote the set of  $k$  such that  $\mathbf{V}_k \neq \mathbf{U}_k$ . Then inequality (9) becomes

$$p(\mathbf{V} \rightarrow \mathbf{U}) \leq \left(\frac{\rho}{8}\right)^{-n_t n_k \delta} \prod_{k \in \eta} |\det(\mathbf{V}_k - \mathbf{U}_k)|^{-2n_k} \quad (10)$$

where  $\delta$  is the size of  $\eta(\mathbf{V}, \mathbf{U})$ , i.e., the block Hamming distance between  $\mathbf{V}$  and  $\mathbf{U}$ . To minimize the pairwise error probability, it is needed to maximize the minimum block Hamming distance,  $\delta_{\min}$ , and the minimum product of squared determinant distance,  $\prod D^2$ , where the minimum block Hamming distance is defined by

$$\delta_{\min} \equiv \min_{\mathbf{U}} |\eta(\mathbf{V}, \mathbf{U})|,$$

the minimum product of squared determinant distance is defined by

$$\prod D^2 \equiv \min_{\mathbf{U}} \left( \prod_{k \in \eta(\mathbf{V}, \mathbf{U})} D^2(\mathbf{V}_k, \mathbf{U}_k) \right),$$

and the determinant distance is defined by

$$D(\mathbf{V}_k, \mathbf{U}_k) \equiv |\det(\mathbf{V}_k - \mathbf{U}_k)|^{\frac{1}{n_k}}.$$

## III. MULTIPLE TRELLIS CODED DUSTM

### A. Code Construction

Consider the multiple trellis coded (MTC) DUSTM which employs a multiple trellis code in place of a trellis code in Fig. 1. Information bits are encoded by a rate  $mb/m(b+1)$  multiple trellis code where  $m$  is the multiplicity [7]. Let  $(L; u_1, u_2, \dots, u_{n_t})$  denote the unitary space-time constellation,  $\mathcal{V} = \{\mathbf{I}, \mathbf{V}_1, \mathbf{V}_1^2, \dots, \mathbf{V}_1^{L-1}\} = \{\mathbf{I}, \mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_{L-1}\}$  (i.e., cyclic group constellation), with

$$\mathbf{V}_l = \text{diag}\left(e^{j\frac{2\pi u_1}{L}}, e^{j\frac{2\pi u_2}{L}}, \dots, e^{j\frac{2\pi u_{n_t}}{L}}\right)$$

and  $\mathbf{V}_l = \mathbf{V}_l^i$  where  $L$  is the cardinality and  $u_1, u_2, \dots, u_{n_t}$  are selected from the positive odd integer set  $\{1, 3, \dots, L-1\}$  in order to maximize the minimum determinant distance,  $D_{\min} = \min_{k \neq k'} D(\mathbf{V}_k, \mathbf{V}_{k'})$ .

Set partitioning for the MTC DUSTM is same as that for conventional multiple trellis coded modulation (MTCM) except that the elements of signal set are not a symbol but  $T$  symbols. Let  $A_0$  denote the complete constellation, i.e., signal points  $0, 1, 2, \dots, L-1$ , and  $A_0 \otimes A_0$  denote a two-fold ordered Cartesian product of  $A_0$  with itself where the ordered Cartesian product means the concatenation of corresponding

elements in the two sets forming the product [7]. The partitioning is as follows.

The first step is to partition  $A_0 \otimes A_0$  into  $L$  signal sets  $A_0 \otimes B_i$ ,  $i=0, 1, \dots, L-1$ , where the  $j$ th element,  $j=0, 1, \dots, L-1$ , of  $B_i$  is  $nj \oplus i$  and  $\oplus$  is modulo  $L$ . The  $j$ th 2-tuple signal points from  $A_0 \otimes B_i$  are the ordered pair  $(j, nj \oplus i)$  where  $n$  is chosen to maximize the minimum product of squared determinant distance  $\prod D^2$  with  $\delta_{\min} = m$  within the partitioned set. Let  $n^*$  denote optimum  $n$ . Then the  $n^*$  for  $\delta_{\min} = 2$  is given as (11) at the bottom of this page. The  $n^*$  is 1, 3 for  $L=8$  and 3, 5 for  $L=16$  from (11).

The second step is an odd-even split of the first level partitioning. The third and succeeding steps are identical to the second step.

Fig. 2 shows the set partitioning of  $(8; [1, 3])$  for multiple ( $m=2$ ) trellis code with  $n^*=3$ . At the first level of the set partitioning, the sets partitioned from the set  $A_0 \otimes A_0$  are given as follows:

$$A_0 \otimes B_0 = \begin{bmatrix} 00 \\ 13 \\ 26 \\ 44 \\ 57 \\ 62 \\ 75 \end{bmatrix}, A_0 \otimes B_2 = \begin{bmatrix} 02 \\ 15 \\ 20 \\ 33 \\ 46 \\ 51 \\ 77 \end{bmatrix}, A_0 \otimes B_4 = \begin{bmatrix} 04 \\ 17 \\ 22 \\ 35 \\ 40 \\ 53 \\ 66 \\ 71 \end{bmatrix}, A_0 \otimes B_6 = \begin{bmatrix} 06 \\ 11 \\ 24 \\ 37 \\ 42 \\ 55 \\ 60 \\ 73 \end{bmatrix}.$$

Since the sets  $A_0 \otimes B_i$ ,  $i=0, 2, 4, 6$ , have the largest  $\prod D^2$  in  $A_0 \otimes A_0$ , sets  $A_0 \otimes B_i$ ,  $i=1, 3, 5, 7$ , are excluded from the set partitioning. If higher data rate is required, sets  $A_0 \otimes B_i$ ,  $i=1, 3, 5, 7$ , may be included in the set partitioning with sacrificing good distance property. From the sets of the first level, the sets of the second level are given as follows:

$$C_0 \otimes D_{00} = \begin{bmatrix} 00 \\ 24 \\ 44 \\ 62 \end{bmatrix}, C_1 \otimes D_{01} = \begin{bmatrix} 13 \\ 31 \\ 57 \\ 75 \end{bmatrix}, C_0 \otimes D_{04} = \begin{bmatrix} 02 \\ 20 \\ 46 \\ 64 \end{bmatrix}, C_1 \otimes D_{01} = \begin{bmatrix} 15 \\ 33 \\ 51 \\ 77 \end{bmatrix},$$

$$C_0 \otimes D_{06} = \begin{bmatrix} 04 \\ 22 \\ 40 \\ 66 \end{bmatrix}, C_1 \otimes D_{01} = \begin{bmatrix} 17 \\ 35 \\ 53 \\ 71 \end{bmatrix}, C_0 \otimes D_{06} = \begin{bmatrix} 06 \\ 24 \\ 42 \\ 60 \end{bmatrix}, C_1 \otimes D_{01} = \begin{bmatrix} 11 \\ 37 \\ 55 \\ 73 \end{bmatrix}.$$

Similarly, from the sets of the second level, the sets of the third level are given as follows:

$$E_0 \otimes F_{00} = \begin{bmatrix} 00 \\ 44 \end{bmatrix}, E_1 \otimes F_{26} = \begin{bmatrix} 26 \\ 62 \end{bmatrix}, E_0 \otimes F_{04} = \begin{bmatrix} 13 \\ 57 \end{bmatrix}, E_1 \otimes F_{31} = \begin{bmatrix} 31 \\ 75 \end{bmatrix},$$

$$E_0 \otimes F_{06} = \begin{bmatrix} 02 \\ 46 \end{bmatrix}, E_1 \otimes F_{20} = \begin{bmatrix} 20 \\ 64 \end{bmatrix}, E_0 \otimes F_{01} = \begin{bmatrix} 15 \\ 51 \end{bmatrix}, E_1 \otimes F_{33} = \begin{bmatrix} 33 \\ 77 \end{bmatrix},$$

$$E_0 \otimes F_{06} = \begin{bmatrix} 04 \\ 40 \end{bmatrix}, E_1 \otimes F_{22} = \begin{bmatrix} 22 \\ 66 \end{bmatrix}, E_0 \otimes F_{01} = \begin{bmatrix} 17 \\ 53 \end{bmatrix}, E_1 \otimes F_{35} = \begin{bmatrix} 35 \\ 71 \end{bmatrix},$$

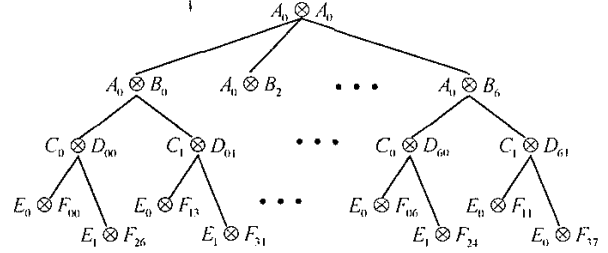
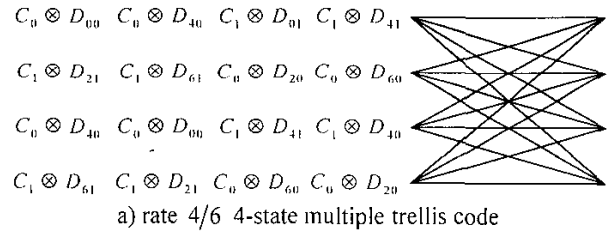


Fig. 2. Set partitioning of  $(8; [1, 3])$  for multiple ( $m=2$ ) trellis code with  $n^*=3$ .

$$E_0 \otimes F_{00} = \begin{bmatrix} 06 \\ 42 \end{bmatrix}, E_1 \otimes F_{24} = \begin{bmatrix} 24 \\ 60 \end{bmatrix}, E_0 \otimes F_{01} = \begin{bmatrix} 11 \\ 55 \end{bmatrix}, E_1 \otimes F_{37} = \begin{bmatrix} 37 \\ 73 \end{bmatrix}.$$

Based on the set partitioning described above, the multiple trellis codes are constructed as follows. All elements in the constellation should be equally probable. The  $2^m$  branches departing from or converging to a state are assigned with elements from one of  $\{A_0 \otimes B_0, A_0 \otimes B_4\}$  and  $\{A_0 \otimes B_2, A_0 \otimes B_6\}$ . The branches departing from or converging to adjacent states are assigned with elements from the other. While minimum product of squared determinant distance  $\prod D^2$  in the sets of the first level is equal to each other,  $\delta_{\min}$  having the  $\prod D^2$  in  $\{A_0 \otimes B_0, A_0 \otimes B_4\}$  and  $\{A_0 \otimes B_2, A_0 \otimes B_6\}$  is larger than that in  $\{A_0 \otimes B_0, A_0 \otimes B_2\}$  and  $\{A_0 \otimes B_4, A_0 \otimes B_6\}$ . Since the error path between the parallel branches becomes the shortest error event path whose Hamming distance is  $\delta_{\min}$ , parallel branches are assigned with elements from the set having the largest product of squared determinant distance at the lowest level of set partitioning. Hence the multiplicity of multiple trellis code guarantees the same  $\delta_{\min}$  as that of trellis code without parallel branches.

Fig. 3 shows the multiple trellis codes for  $(8; [1, 3])$ . Fig. 3a) and Fig. 3b) show the rate 4/6 4-state and 8-state multiple trellis code for  $(8; [1, 3])$ , respectively. Rate 6/8 multiple trellis codes for  $(16; [1, 7])$  are constructed by the same method by which multiple trellis for  $(8; [1, 3])$ .



$$n^* = \arg \max_n \prod D^2 = \arg \max_{n=1,3,\dots,L/2-1} \left\{ \min_{m=1,2,\dots,L/2-1} 16 \sin\left(\frac{mu_1\pi}{L}\right) \sin\left(\frac{nm_1\pi}{L}\right) \sin\left(\frac{mu_2\pi}{L}\right) \sin\left(\frac{nm_2\pi}{L}\right) \right\}. \quad (11)$$

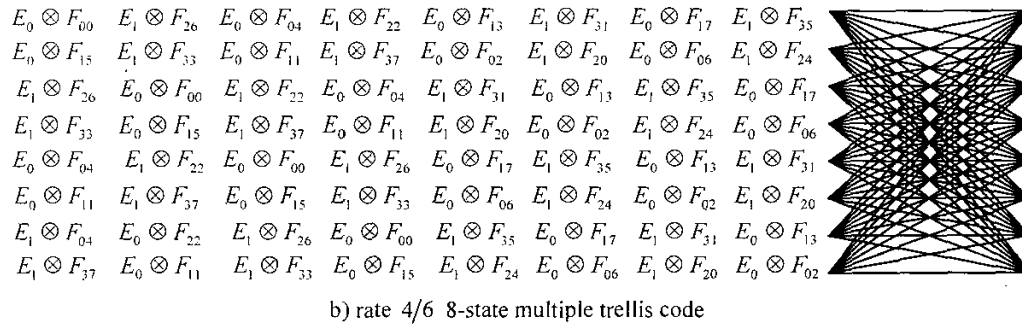


Fig. 3. Multiple trellis codes for  $(8; [1, 3])$ .

*B. Computational Complexity*

Since the number of branches per a transition in the trellis increases exponentially as spectral efficiency increases in a multiple trellis code, it has much higher computational complexity than a trellis code.

The computational complexity is decreased by reducing the number of branches as follows. Compute and store the branch metrics of (7) for all elements in the unitary space-time constellation  $\mathcal{V}$  at each block. Find most-likely candidates within each set assigned to the parallel branches, and then, make maximum likelihood decision among only the most-likely candidates [8]. The sets assigned to the parallel branches are the ordered Cartesian product of  $\mathcal{V}$ . Once the branch metrics for the elements of each set are computed and stored, the branch metrics for repeated elements need not to be calculated again.

The computational complexity is evaluated in terms of the number of operations such as addition, multiplication, and comparison. Let  $M_b$  denote the number of operations needed to compute a branch metric. For example,  $M_b = 10$  for  $(8; [1, 3])$ , i.e., 8 multiplications and 2 additions are performed. Let  $M_t$  denote the number of operations needed to decode a symbol. Then, the number of operations in the MTC DUSTM is given by

$$M_t = \frac{2^{b+1} \times m \times M_b + 2 \times (2^{mb+1} + 2^b \times S)}{m \times n_r} \quad (12)$$

where  $S$  is the number of states. The number of operations in the trellis coded DUSTM is given by

$$M_t = \frac{2^{b-1} \times M_b + 2 \times 2^b \times S}{n_r} \quad (13)$$

Table I shows the number of operations in the MTC DUSTM compared to that in the trellis coded DUSTM. It is shown that the number of operations in the former is comparable to that in the latter.

TABLE I  
Number of operations in the MTC DUSTM

R bits/s/Hz	Type of coded DUSTM	$M_t$
1	4-state MTC $(8; [1, 3])$	64
	8-state MTC $(8; [1, 3])$	72
	4-state trellis coded $(8; [1, 3])$	56
	8-state trellis coded $(8; [1, 3])$	72
1.5	8-state MTC $(16; [1, 7])$	176
	16-state MTC $(16; [1, 7])$	208
	8-state trellis coded $(16; [1, 7])$	114
	16-state trellis coded $(16; [1, 7])$	208

IV. NUMERICAL RESULTS

Table II shows the minimum block Hamming distance and minimum product of squared determinant distance of the MTC DUSTM compared to that of the trellis coded DUSTM. While the minimum block Hamming distance of the former is same as that of the latter, the minimum product of squared determinant distance of the former is larger than that of the latter.

Fig. 4 shows the BER of the MTC DUSTM compared to that of the uncoded DUSTM and trellis coded DUSTM for 2 transmit antennas and 1 receive antenna. Fig. 4a) shows the BER of the MTC DUSTM for spectral efficiency  $R = 1$  bit/s/Hz. In Fig 4a) it is shown that the 4-state and 8-state MTC DUSTM give 0.9 dB and 1.0 dB gain over the 4-state and the 8-state trellis coded DUSTM at a BER of  $10^{-5}$ , respectively. Fig. 4b) shows the BER of the MTC DUSTM for spectral efficiency  $R = 1.5$  bits/s/Hz. In Fig. 4b) it is shown that the 8-state and 16-state MTC DUSTM give 2.5 dB and 0.7 dB gain over the 8-state and 16-state trellis coded DUSTM at a BER of  $10^{-5}$ , respectively.

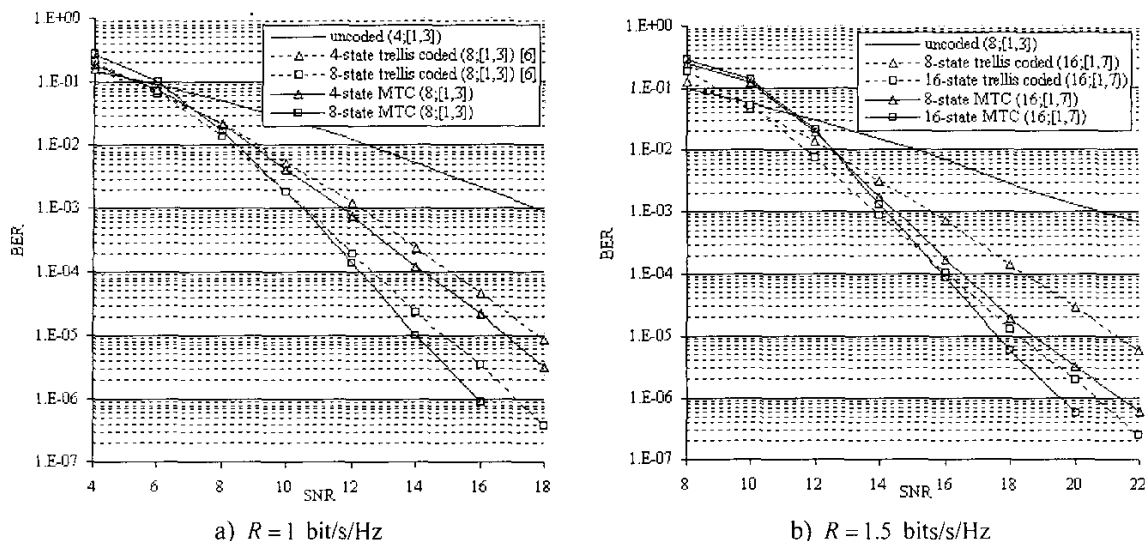


Fig. 4. BER of the MTC DUSTM compared to the uncoded DUSTM and trellis coded DUSTM for 2 transmit antennas and 1 receive antenna.

### V. CONCLUSIONS

In this paper, the multiple trellis coded DUSTM is considered for multiple-antenna systems without channel state information. The construction of multiple trellis codes for DUSTM is described, and several codes constructed are provided. The computational complexity is considered. The BER of the MTC DUSTM is evaluated for various numbers of states through computer simulation. It is shown that the MTC DUSTM achieves smaller BER than the trellis coded DUSTM without much increase of the computational complexity for the same spectral efficiency.

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TABLE II

Minimum block Hamming distance and minimum product of squared determinant distance of the MTC DUSTM

$R$	Type of coded DUSTM	$\delta_{\min}$	$\prod D^2$
1	4-state MTC (8;[1,3])	2	4
	8-state MTC (8;[1,3])	2	16
	4-state trellis coded (8;[1,3])	2	2.828
	8-state trellis coded (8;[1,3])	2	8
1.5	8-state MTC (16;[1,7])	2	2
	16-state MTC (16;[1,7])	2	4
	8-state trellis coded (16;[1,7])	2	1.083
	16-state trellis coded (16;[1,7])	2	1.537

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