Outage Analysis of Cognitive Relay Networks over Double Rayleigh Fading Channels

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Abstract—In this paper, the underlay cognitive relay network over double Rayleigh fading channels is investigated. For this network, we consider two cases: limited power case and unlimited power case. In the limited power case, the power of secondary user is constrained. On the other hand, the power of secondary user is not constrained in the unlimited power case. We derive the exact and approximate outage probabilities for two cases. It is shown that for limited power case, the approximate outage probability approaches the exact outage probability as the maximum tolerable interference level increases. Also, it is shown that for unlimited power case, the approximate outage probability is similar to the exact outage probability.

Index terms — Cognitive radio network, relay, double Rayleigh fading, outage probability.

I. INTRODUCTION

In the cognitive radio networks, the secondary users are allowed to access the primary user's spectrum using underlay, overlay and interweave approaches [1]. In particular, for underlay cognitive networks, the secondary user is allowed to share the primary user's spectrum as long as the interference caused by the secondary user is below the maximum tolerable interference level at the primary user receiver [2].

Double (cascaded) Rayleigh fading channel model has been proposed which provides a realistic description of an inter-vehicular channel where two Rayleigh fading processes are assumed to be generated by independent groups of scatterers around the two mobile terminals [3]. In [4], the authors investigate the performance of amplify-and-forward relaying for an inter-vehicular cooperative scheme assisted by either a roadside access point or another vehicle that acts as a relay. In [5], the cooperative diversity with relay selection over double Rayleigh fading channels is investigated.

Recently, several works have investigated cognitive relay networks over various fading channels based on the underlay approach [6]. In [7], the exact outage probability of an underlay cognitive network using decode-and-forward (DF) relaying with best relay selection over Rayleigh fading channels has been investigated. In [8], the outage probability performance of DF cognitive relay networks over Nakagami-m fading channels has been studied. However, the underlay cognitive network over double Rayleigh fading channels has not been investigated.

In this paper, we investigate the underlay cognitive relay network where channels for the secondary network are modeled as double Rayleigh fading channels. For this network, we consider two cases: limited power case and unlimited power case. In the limited power case, the power of the secondary user is constrained. On the other hand, the power of the secondary user is not constrained in the unlimited power case. We derive the exact and approximate outage probabilities for two cases. Numerical results verify the validity of our theoretical analysis by comparison with Monte Carlo simulation.

The rest of this paper is organized as follows. In Section II, we describe the system model and the relay selection for the underlay cognitive relay network. In Section III, we derive the exact and the approximate outage probabilities for two power cases. In Section IV, the numerical results verify the validity of our theoretical analysis by comparison between the analytical results and Monte Carlo simulation. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

Consider an underlay cognitive relay network as shown in Fig. 1. In this network, there are one primary destination, one secondary source, one secondary destination, and K secondary relays. Assume that the primary source does not interfere at secondary relays and the secondary destination as the primary
source is far away from secondary relays and the secondary destination. It is assumed that secondary users utilize the primary user’s spectrum if the interference at the primary destination remains below the predetermined threshold. Assume that the primary destination occupies multiple spectra and each relay uses different spectrum among the primary spectra. Suppose that each node has a single antenna. Assume that interference channels, the channels from secondary users to primary destination, are quasi-static flat Rayleigh fading channels and channels for the secondary network are quasi-static flat double Rayleigh fading channels. This assumption is reasonable as the system model considered in this paper is the simplified model of the cognitive vehicular network where a primary destination is an infrastructure and secondary users are vehicles. It is assumed that there is no direct transmission between the secondary source and the secondary destination.

Let $PD, SS, SD$, and $SR_k$ denote the primary destination, the secondary source, the secondary destination, and the $k$th secondary relay. Assume that the communication between $SS$ and $SD$ is performed in two phases under the half-duplex constraint. In the first phase, $SS$ transmits the signal to the best relay $SR_k^*$ through the secondary spectrum for $SR_k$. In the second phase, $SR_k$ decodes and forwards the re-encoded signal to $SD$ through the secondary spectrum for $SR_k$.

Let $h_{S,k}, h_{k,D}, h_{k,P}$, and $h_{S,k,P}$ denote the channel coefficients from $SS$ to $SR_k$, from $SR_k$ to $SD$, from $SR_k$ to $PD$, and from $SS$ to $PD$ through the secondary spectrum for $SR_k$, respectively. From the assumption that channels for the secondary network are double Rayleigh fading channels, $h_{S,k}$ and $h_{k,D}$ are modeled as $h_{S,k} = h_{S,k,1} h_{S,k,2}$ and $h_{k,D} = h_{k,D,1} h_{k,D,2}$, respectively. Assume that $h_{S,k,P}, h_{k,P}, h_{S,k,1}, h_{S,k,2}, h_{k,D,1}$, and $h_{k,D,2}$ are independent zero-mean circularly symmetric complex Gaussian random variables with variances $1/\lambda_{S,k,P}, 1/\lambda_{k,P}, 1/\lambda_{k,1}, 1/\lambda_{k,2}, 1/\lambda_{k,3}$, and $1/\lambda_{k,4}$, respectively. Since the proactive DF scheme is used, a relay selection criterion is defined as follows [9]:

$$k^* = \arg \max_k \left\{ P_{S_k} |h_{S,k}|^2, P_k |h_{k,D}|^2 \right\}$$  \hspace{1cm} (1)

where $P_{S_k}$ is the transmit power of $SS$ through the secondary spectrum for $SR_k$ and $P_k$ is the transmit power of $SR_k$.

### III. Outage Probability Derivation

We consider two cases: limited power case and unlimited power case. In the limited power case, the power constraint exists for $SS$ and $SR_k$. In the unlimited power case, there is no power constraint at $SS$ and $SR_k$.

#### A. Limited Power Case

In the limited power case, the transmit power of $SS$ and $SR_k$ is constrained as follows:

$$P_{S_k}^l \leq \min \left\{ \frac{I}{|h_{S,k}|^2}, \bar{P} \right\}$$  \hspace{1cm} (2)

where $\bar{P}$ is the maximum tolerable interference at $PD$ and $\bar{P}$ is the maximum transmit power. The outage probability is given by (4) at the top of the next page where $N_0$ is the noise variance, $R$ is the target rate, and $P_{th} = N_0(2R - 1)$.

When the outage probability is defined as $p_{out,l} = \prod_{k=1}^{K} p_{out,k}^l$, $p_{out,k}^l$ is given by

$$p_{out,k}^l = \Pr \left[ \min \left( P_{S_k}^l |h_{S,k}|^2, P_k^l |h_{k,D}|^2 \right) \leq P_{th} \right]$$

Let $h_{S,k}^l, h_{k,D}^l, P_k^l, \bar{P}$, and $P_{th}$ be the transmit power of $k$, $P_{th}$ is given by

$$P_{th} = \Pr \left[ \min (U_k, V_k) \leq P_{th} \right]$$

$$= 1 - \Pr [U_k > P_{th}] \Pr [V_k > P_{th}]$$

$$= F_{U_k}(P_{th}) + F_{V_k}(P_{th}) - F_{U_k}(P_{th}) F_{V_k}(P_{th})$$  \hspace{1cm} (5)

where $F_{U_k}(\cdot)$ and $F_{V_k}(\cdot)$ are CDF of $U_k$ and $V_k$, respectively. From (2), $F_{U_k}(x)$ is given by

$$F_{U_k}(x) = \Pr \left[ \min \left( \frac{I}{|h_{S,k}|^2}, \bar{P} \right)^2 \leq x \right]$$

$$= \alpha (x) + \beta (x)$$  \hspace{1cm} (6)

where $\alpha (x) = \Pr [\bar{P}|h_{S,k}|^2 \leq x, I/|h_{S,k}|^2 > \bar{P}]$ and $\beta (x) = \Pr [I/|h_{S,k}|^2 \leq x, I/|h_{S,k}|^2 \leq \bar{P}]$. The CDF of $|h_{S,k}|^2$ is given by

$$F_{|h_{S,k}|^2}(x) = 1 - \sqrt{4\lambda_{k,1} \lambda_{k,2} x} K_1 \left( \sqrt{4\lambda_{k,1} \lambda_{k,2} x} \right)$$  \hspace{1cm} (7)

where $K_1(\cdot)$ is the modified Bessel function of the second kind.

Since $|h_{S,k}|^2$ and $|h_{S,k,P}|^2$ are independent and $|h_{S,k}|^2$ is exponentially distributed [11], $\alpha (x)$ is given by

$$\alpha (x) = F_{|h_{S,k}|^2} \left( \frac{x}{\bar{P}} \right) F_{h_{S,k,P}} \left( \frac{I}{\bar{P}} \right)$$

$$= \left( 1 - \sqrt{4\lambda_{k,1} \lambda_{k,2} x} K_1 \left( \sqrt{4\lambda_{k,1} \lambda_{k,2} x} \right) \right) \times \left( 1 - \exp \left( -\frac{\lambda_{S,k,P} I}{\bar{P}} \right) \right)$$  \hspace{1cm} (8)

In (6), $\beta (x)$ is given by at the top of the next page. Putting (8) and (9) into (6), $F_{U_k}(x)$ can be obtained. Similarly, $F_{V_k}(x)$ can be obtained. Accordingly, by using $F_{U_k}(x)$ and $F_{V_k}(x)$, the exact outage probability can be obtained.
\[ P_{\text{out},l} = \Pr \left[ \frac{1}{2} \min \left\{ \log_2 \left( 1 + \frac{P_{S_k} |h_{S,k}|^2}{N_0} \right), \log_2 \left( 1 + \frac{P_{k} |h_{k,D}|^2}{N_0} \right) \right\} \leq R \right] \]
\[ = \Pr \left[ \min \left( P_{S_k} |h_{S,k}|^2, P_{k} |h_{k,D}|^2 \right) \leq P_{th} \right] \]

(4)

\[ \beta (x) = \int_0^\infty f_{j_{S_k,p}^2} (y) \int_0^{xy/I} f_{j_{S_k,p}^2} (z) \, dz \, dy \]
\[ = \int_0^\infty \frac{\lambda_{S_k,p} \exp (-\lambda_{S_k,p} y)}{I} \left( 1 - \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D} x}{I}} K_1 \left( \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D}}{I}} \right) \right) \, dy \]
\[ = \exp \left( -\frac{\lambda_{S_k,p} I}{P} \right) - \lambda_{S_k,p} \lambda_{k,D} \int_0^\infty \exp (-\lambda_{k,D} w) \int_0^\infty \exp \left( -\frac{w I \lambda_{S_k,p} + \lambda_{k,D} x}{w I} \right) \, dy \, dw \]
\[ = \exp \left( -\frac{\lambda_{S_k,p} I}{P} \right) \left( 1 - \lambda_{k,D} \int_0^\infty \exp (-\lambda_{k,D} w) \exp \left( -\frac{w I}{w I + \lambda_{k,D} x / \lambda_{S_k,p} I} \right) \, dw \right) \]

(9)

When \( \lambda_{k,1} x \ll \lambda_{S_k,p} I \), \( \beta (x) \) can be approximated as

\[ \beta (x) \simeq \left( 1 - \lambda_{k,D} \int_0^\infty \exp (-\lambda_{k,D} w) \exp \left( -\frac{\lambda_{k,1} x}{w P} \right) \, dw \right) \]
\[ \times \exp \left( -\frac{\lambda_{S_k,p} I}{P} \right) \]
\[ = \left( 1 - \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D} x}{P}} K_1 \left( \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D}}{P}} \right) \right) \]
\[ \times \exp \left( -\frac{\lambda_{S_k,p} I}{P} \right) \]

(10)

Putting (8) and (9) into (6), \( F_{U_k} (x) \) is approximated as

\[ F_{U_k} (x) \simeq 1 - \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D} x}{P}} K_1 \left( \sqrt[4]{\frac{4 \lambda_{S_k,p} \lambda_{k,D}}{P}} \right) \]

(11)

Similarly, \( F_{V_k} (x) \) can be approximated. Therefore, the outage probability is approximated as at the top of the next page.

**B. Unlimited Power Case**

In the unlimited power case, the transmit power of \( SS \) and \( SR_k \) is given by

\[ P_{S_k}^u = \frac{I}{|h_{S,k}|^2}, \]
\[ P_{k}^u = \frac{I}{|h_{k,D}|^2}. \]

(13)

When the outage probability is defined as \( P_{\text{out},u} = \prod_{k=1}^K P_{k}^\text{out,u} \), \( P_{k}^\text{out,u} \) is given by

\[ P_{k}^\text{out,u} = \Pr \left[ \min \left( \left| h_{S_k,k} \right|^2, \left| h_{k,D} \right|^2 \right) \leq \eta \right] \]
\[ = \Pr \left[ \min \left\{ M_k, N_k \right\} \leq \eta \right] \]
\[ = F_{M_k} (\eta) + F_{N_k} (\eta) - F_{M_k} (\eta) F_{N_k} (\eta) \]
\[ \text{where } \eta = N_0 (2^R - 1) / I. \]

The CDF of \( M_k \) is given by

\[ F_{M_k} (x) = \Pr \left[ \left| h_{S_k,k} \right|^2 \leq x \right] \]
\[ = \int_0^x \Pr \left[ \left| h_{S_k,k} \right|^2 \leq y \right] f_{j_{S_k,k}^2} (y) \, dy \]
\[ = \int_0^\infty F_{j_{S_k,k}^2} (xy) f_{j_{S_k,k}^2} (y) \, dy \]
\[ = \rho_k \exp (\rho_k \eta) \Gamma (0, \rho_k \eta) \]
\[ \rho_k = \lambda_{k,1} \lambda_{S_k,p} / |h_{S_k,p}|^2 \text{ and } \Gamma (\cdot, \cdot) \text{ is incomplete gamma function.} \]

Similarly, \( F_{N_k} (x) \) can be obtained. Accordingly, using \( F_{M_k} (x) \) and \( F_{N_k} (x) \), \( P_{k}^\text{out,u} \) is given by

\[ P_{k}^\text{out,u} = \rho_k \eta \exp (\rho_k \eta) \Gamma (0, \rho_k \eta) + \kappa_k \eta \exp (\kappa_k \eta) \Gamma (0, \kappa_k \eta) \]
\[ - \rho_k \kappa_k \eta^2 \exp ((\rho_k + \kappa_k) \eta) \Gamma (0, \rho_k \eta) \Gamma (0, \kappa_k \eta), \]
\[ \text{where } \kappa_k = \lambda_{k,3} \lambda_{S_k,p} / |h_{k,D}|. \]

By using (17), the exact outage probability can be obtained.

We have [12]

\[ \Gamma (0, x) = -E_1 (-x) \]
\[ = - \left( \gamma + \ln |x| + \sum_{k=1}^\infty \frac{(-x)^k}{k k!} \right) \]
\[ \simeq - \gamma - \ln |x| + x, \text{ as } x \to 0, \]

(18)
\[ p_{\text{out},l} \simeq \prod_{k=1}^{K} \left( 1 - \frac{4P_{\text{th}}}{P} \sqrt{\frac{\lambda_{k,1}\lambda_{k,2}\lambda_{k,3}\lambda_{k,4}}{P}} K_1 \left( \sqrt{\frac{4\lambda_{k,1}\lambda_{k,2}P_{\text{th}}}{P}} \right) K_1 \left( \sqrt{\frac{4\lambda_{k,3}\lambda_{k,4}P_{\text{th}}}{P}} \right) \right) \] (12)

where \( E_i(x) \) is exponential integral and \( \gamma \) is Euler-Mascheroni constant. Accordingly, \( P_{\text{out},u}^k \) is approximated as

\[
P_{\text{out},u}^k \simeq \rho_k \eta \exp(\rho_k \eta \eta - \ln(\rho_k \eta) - \gamma) + \kappa_k \eta \exp(\kappa_k \eta \eta - \ln(\kappa_k \eta) - \gamma)
- \rho_k \kappa_k \eta^2 \exp((\rho_k + \kappa_k) \eta)
\times (\rho_k \eta - \ln(\rho_k \eta) - \gamma)(\kappa_k \eta - \ln(\kappa_k \eta) - \gamma).
\] (19)

By using (19), the approximate of \( P_{\text{out},u}^k \) can be obtained.

IV. SIMULATION RESULTS

Consider an underlay cognitive radio network with \( K \) relays. Suppose that the transmit data rate, \( R \), is 1 bps/Hz and \( \lambda_{S,k} = \lambda_{k,D} = 1 \) for \( k = 1, \ldots, K \). We also suppose that \( \lambda_{S,k,P} = \lambda_{k,P} = \lambda \) for \( k = 1, \ldots, K \).

Fig. 2 shows exact and approximate outage probabilities versus \( I/N_0 \) using different \( K \) for the limited power case. The exact outage probabilities are plotted by using Monte Carlo simulation and the approximate outage probabilities are plotted according to (12). It is shown that the approximate outage probabilities approaches the exact outage probabilities as \( I/N_0 \) increases. Note that the approximate outage probabilities are not affected by \( I/N_0 \).

In Fig. 3, the outage probabilities for the unlimited power case are presented. For comparison, we plot the outage probability for the system that the channels of the secondary network are modeled as Rayleigh fading. It is shown that the outage performance of the secondary network over double Rayleigh fading is inferior to that over Rayleigh fading. Fig. 3 also shows that a full diversity order can be achieved.

Fig. 4 shows the exact and approximate outage probabilities versus \( I/N_0 \) using different \( K \) for the unlimited power case. It is shown that the approximate outage probability is very close to the exact outage probability.

Fig. 5 shows the exact and approximate outage probabilities versus \( I/N_0 \) using different \( \lambda \) for the unlimited power case. It is shown that the approximate outage probability is similar
to the exact outage probability in high $\bar{I}/N_0$ region. Since we assume that $\rho_k \eta \to 0$ and $\kappa_k \eta \to 0$ to obtain (19), the exact and approximate outage probabilities does not match well as $\lambda \to 0$ and $\bar{I} \to 0$.

V. CONCLUSIONS

In this paper, we consider the underlay cognitive relay network where channels for the secondary network are modeled as double Rayleigh fading channels. For two power cases, limited and unlimited power cases, we derive the exact and approximate outage probabilities of the secondary user. It is shown that for limited power case, the approximate outage probability approaches the exact outage probability as the maximum tolerable interference increases. Also, it is shown that for unlimited power case, the approximate outage probability is similar to the exact outage probability.

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