Outage Probability of a Two-Way Full-Duplex Relay Network over Rayleigh Fading Channels

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Abstract

This paper analyzes the performance of a two-way full-duplex relay network over Rayleigh fading channels. We derive the exact- and approximate-form expressions for the outage probability of the two-way full-duplex relay network over Rayleigh fading channels. The validity of our analytical results is verified by a comparison with simulation results. It is shown that the analytical results well match the simulation results of the outage probability.

Keywords: Two-way, full-duplex, relay, Rayleigh fading channels.

1. Introduction

Two-way relay network provides improved spectral efficiency by using either superposition coding or physical layer network coding at the relays compared to conventional one-way relay networks [1]-[3].

Full-duplex relay network, where the transmission and the reception at the relay occur at the same time on the same channel, can achieve up to double the capacity of a half-duplex relay network [4]-[6].

In this paper, we investigate the two-way full-duplex relay network over Rayleigh fading channels. We drive the exact- and approximate-form expressions for the outage probability of the two-way full-duplex relay network. Analytical formulas are verified by simulations.

2. System Model

The nodes $a$ and $b$ exchange information with the help of the amplify-and-forward relay $r$. Assume that and there is no direct path between the nodes $a$ and $b$.

Assume that the users $a$ and $b$ communicate with each other in two phases: the multiple-access (MA) and broadcast (BC) phases. In the MA phase, the nodes $a$ and $b$ transmit their signals to the relay $r$ simultaneously, and then, the relay $r$ receives not only the transmitted signals from the nodes $a$ and $b$ but also the loop interference. The received signal at the relay $r$ is given by

$$ y_r(t) = \sqrt{E_i} h_{ar}(t)x_a(t) + \sqrt{E_i} h_{br}(t)x_b(t) + \sqrt{E_i} h_{rr}(t)x_r(t) + n_r(t) $$

(1)

where $h_{ij}(t), \ i, j \in \{a, b, r\}$, is the channel coefficient from the node $i$ to the node $j$ which an independent zero-mean circularly symmetric complex Gaussian random variable with the variance $\lambda_{ij}, x_i(t)$ is the transmit signal from the node $i$, $E_i$ is the transmit energy from the node $i$, and $n_i(t)$ is the additive white Gaussian noise at the node $i$. The third term of the right hand side of (1) represents the loop interference from the relay $r$ itself.

The relay $r$ subtracts an estimate of the loop interference from its received signal.

$$ \tilde{y}_r(t) = y_r(t) - \sqrt{E_i} \tilde{h}_{rr}(t)x_r(t) $$

$$ = \sqrt{E_i} h_{ar}(t)x_a(t) + \sqrt{E_i} h_{br}(t)x_b(t) + \sqrt{E_i} \Delta h_{rr}(t)x_r(t) + n_r(t) $$

(2)

where $\tilde{h}_{rr}(t) \sim \mathcal{CN}(0, \mu_{rr})$ is the estimate of the loop interference channel and $\Delta h_{rr}(t) \sim \mathcal{CN}(0, \nu_{rr})$ is the corresponding estimation error.

In the BC phase, the relay $r$ amplifies its received signal by the factor $\alpha$ which is given by

$$ \alpha(t) = \frac{1}{\sqrt{E_r \| h_{rr}(t-1) \|^2 + E_r \| h_{ar}(t-1) \|^2 + E_r \nu_{rr} + N_0}} $$

(3)

The transmit signal from the relay $r$ is given by
\[ x_r(t) = \alpha(t)\tilde{y}_r(t-1), \quad (4) \]

which is forwarded to the nodes \(a\) and \(b\). The received signals at the nodes \(a\) and \(b\) are given by

\[ y_a(t) = \sqrt{E} h_{a,a}(t)x_r(t) + \sqrt{E} h_{a,b}(t)x_a(t) + n_a(t) \quad (5) \]

\[ y_b(t) = \sqrt{E} h_{b,a}(t)x_r(t) + \sqrt{E} h_{b,b}(t)x_b(t) + n_b(t) \quad (6) \]

The nodes \(a\) and \(b\) subtracts the self-interference and an estimate of the loop interference from its received signal as follows:

\[ \tilde{y}_a(t) = y_a(t) - \alpha(t)\sqrt{E} h_{a,a}(t)h_{a,a}(t-1)x_r(t-1) - \sqrt{E} h_{a,b}(t)x_a(t) \quad (7) \]

\[ \tilde{y}_b(t) = y_b(t) - \alpha(t)\sqrt{E} h_{b,b}(t)h_{b,b}(t-1)x_r(t-1) - \sqrt{E} h_{b,a}(t)x_b(t) \quad (8) \]

where \( \tilde{h}_i(t) \sim \text{CN}(0, \mu_i), i \in \{a, b\} \), is the estimate of the loop interference channel.

### 3. Outage Probability

After self-interference cancellation and loop interference cancellation, the signal-to-noise ratios (SNRs) at the nodes \(a\) and \(b\), \( \gamma_a \) and \( \gamma_b \), can be obtained. From the SNRs at the nodes \(a\) and \(b\), \( \gamma_a \) and \( \gamma_b \), we obtain the outage probability as follows:

\[ P_{\text{out},a}(\gamma_{th}) = \Pr(\gamma_a < \gamma_{th}) \quad (9) \]

\[ P_{\text{out},b}(\gamma_{th}) = \Pr(\gamma_b < \gamma_{th}) \quad (10) \]

where \( \gamma_{th} \) is the predetermined threshold SNR.

### 4. Simulation Results

Consider a two-way full-duplex relay network over Rayleigh fading channels. Suppose that \( \gamma_a = 1\text{dB} \), \( \lambda_{i,j} = 1 \), and \( \mu_{i,j} = \lambda_{i,j} - \nu_{i,j} \) for \( i, j \in \{a, b, r\} \).

Fig. 1 shows the outage probability of the node \(a\) versus SNR for the two-way full-duplex relay network. It is shown that the analytical results well match the simulation results of outage probability. Also, it is shown that the outage probability increases as the variance of the channel estimation error increases.

![Outage probability of the node a versus SNR for the two-way FD relay network.](image)

**Fig. 1.** Outage probability of the node \(a\) versus SNR for the two-way FD relay network.

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### References


