Outage Probability Analysis of ARQ-Based DF Opportunistic Relaying with Outdated CSI

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Abstract—In this paper, the outage performance of automatic repeat request (ARQ) based opportunistic relaying is analyzed when the channel state information (CSI) for relay selection is outdated. The outage probability is derived in terms of correlation between SNR and outdated SNR of the channel from the selected relay to destination. It is shown that simulation results of the outage probability match with its analysis.

Keywords: ARQ, cooperative diversity, opportunistic relaying, outdated CSI.

I. INTRODUCTION

Cooperative diversity is an effective technique to improve the reliability of communications over wireless channels by using relay [1]. For multi-relay systems, opportunistic relaying is an efficient cooperative diversity scheme by achieving full diversity with low system complexity [2]. In opportunistic relaying, the best relay is selected based on the instantaneous channel state information (CSI) of end-to-end paths [2] or paths from the relays to the destination [3]. When a delay constraint is not strict, automatic repeat request (ARQ) can be adopted in opportunistic relaying to further improve the reliability of communications [4].

In practice, the CSI for relay selection is not perfect but outdated because of the channel variations caused by time delay between the relay selection and the transmission by the selected relay. In [5], it is shown that the outage performance of opportunistic relaying is severely degraded when outdated CSI is available. As ARQ-based opportunistic relaying is utilized for environment with fast channel variation [6], the effect of outdated CSI on ARQ-based opportunistic relaying is not negligible and needs to be analyzed which hasn’t considered yet on prior works.

In this paper, we investigate dual-hop DF ARQ-based opportunistic relaying system when outdated CSI is available. A quantitative model of channel outdatedness is introduced. Then, the outage probability for this system is derived in a closed form. It is approximated to a simpler form which is valid at high SNR.

This paper is organized as follows. In section II, the system model is described. In section III, the outage probability of ARQ-based opportunistic relaying is derived and approximated. In section IV, the numerical results are provided. Conclusions are drawn in section V.

II. SYSTEM MODEL

Consider a half-duplex DF dual-hop system with ARQ-based opportunistic relaying which consists of a source, a destination, and K relays, each with a single antenna, as shown in Fig. 1. Assume that there is no direct path between the source and the destination.

The source, s, broadcasts an encoded data to K relays, r_k, k = 1, 2, ..., K, with rate R. Let the decoding set D denote the set of relays that succeed in decoding the received signal from the source. In the decoding set, a relay with the best channel from the relay to the destination is selected to re-encode and transmit the decoded data with rate R to the destination, d, by opportunistic relaying. The destination decodes the received signal from the selected relay and if the decoding is successful, transmits an ACK to all relays. Otherwise, the destination transmits a NACK and a relay with the best channel to the destination is selected to retransmit the same signal until the destination successfully decodes the received signal or the number of ARQ rounds reaches N. Assume that each relay receives an ACK/NACK error-free.
Assume that the channel coefficients from the source to the relay \( r_i, h_k, k = 1, 2, \cdots, K \), are circularly symmetric complex Gaussian random variables with zero-mean and unit variance. Then the received SNR at the relay \( r_i \) is given by
\[
\Gamma_{r_i} = \left| h_{r_i} \right|^2 E_s / N_0,
\]
where \( E_s \) is the energy of the transmit signal and \( N_0 \) is the noise variance at the relay \( r_i \). If the mutual information at the relay \( r_i \) is higher than the rate \( R \), then the relay \( r_i \) succeeds in decoding the signal.

Assume that the channel coefficients from the relay \( r_i \) to the destination at the \( n \)th ARQ round, \( h_{k,r_i}, k = 1, 2, \cdots, K, n = 1, 2, \cdots, N \), are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit variance. Then the SNR of the channel from the relay \( r_i \) to the destination at the \( n \)th ARQ round is given by
\[
\Gamma_{kr_i} = \left| h_{k,r_i} \right|^2 E_s / N_0,
\]
where \( E_s \) is the energy of the transmit signal and \( N_0 \) is the noise variance at the destination. Note that \( \Gamma_{r_i} \) and \( \Gamma_{kr_i} \) have the same mean \( \Gamma = E_s / N_0 \).

Assume that the CSI for relay selection is outdated due to the channel variations during time delay between the moments of relay selection and transmission by the selected relay. Assume that the outdated channel coefficient \( h_{k,r_i}^{(o)} \), \( k = 1, 2, \cdots, K, n = 1, 2, \cdots, N \), has the same distribution as \( h_{k,r_i} \). The outdated SNR for relay selection at the \( n \)th ARQ round is given by
\[
\Gamma_{r_i}^{(o)} = \left| h_{r_i}^{(o)} \right|^2 E_s / N_0
\]
with mean \( \Gamma \).

Then the selected relay at the \( n \)th ARQ round, \( r^* \), is given by
\[
r^* = \arg \max_{r_i} \hat{\Gamma}_{r_i}^{(o)}.
\]  
(1)

According to Jakes’ model, the correlation between the SNR \( \Gamma_{r_i}^{(o)} \) and the outdated SNR \( \hat{\Gamma}_{r_i}^{(o)} \) is given by
\[
\rho = J_0^2 (2\pi f_d \tau)
\]  
(2)

where \( J_0(\cdot) \) is the zeroth-order Bessel function of the first kind, \( f_d \) is the maximum Doppler frequency, and \( \tau \) is the delay between the moments of relay selection by the destination and the transmission by the selected relay [7].

If the mutual information at the destination at the \( n \)th ARQ round, \( n = 1, 2, \cdots, N-1 \), is larger than the rate \( R \), then the destination succeeds in decoding the signal. Otherwise, the destination discards the received signal at \( n \)th ARQ round and attempts to receive and decode the signal at the next ARQ round. An outage is declared when the destination cannot decode the received signal until \( N \) ARQ rounds.

III. OUTAGE PROBABILITY

As the channel from the source to each relay is statistically identical, the probability of decoding failure at the relay \( r_i \) is given by \( 1 - e^{-\nu} \Gamma \). Then, the probability that \( l \) out of \( K \) relays succeed in decoding the received signal from the source is given by [5]
\[
Pr[D = l] = \binom{K}{l} e^{-\nu \Gamma} \left(1 - e^{-\nu \Gamma} \right)^{K-l}
\]  
(3)

where \( [D] \) is the cardinality of \( D \) and \( \gamma_0 = 2^{\beta} - 1 \).

The outage probability is given by
\[
P_{out} = Pr[D = 0] + P_{fail}^{(N)}
\]  
(4)

where \( P_{fail}^{(N)} \) is the probability of decoding failure at the destination until the \( N \)th ARQ round. To obtain \( P_{fail}^{(N)} \), we derive the cdf of the SNR at the destination at the \( N \)th ARQ round.

The outdated received SNR at the destination at the \( n \)th ARQ round is the maximum of \( \Gamma_{kr_i}^{(o)} \), \( r_i \in D \). Its cdf conditioned on \( [D] \) is given by
\[
F_{\Gamma_{kr_i}^{(o)}}(\hat{\gamma} | l) = \left(1 - e^{-\hat{\gamma} / \Gamma} \right)^l
\]  
(5)

By differentiating (5) with respect to \( \hat{\gamma} \) and applying binomial expansion, the pdf of \( \Gamma_{kr_i}^{(o)} \) conditioned on \( [D] \) is given by
\[
f_{\Gamma_{kr_i}^{(o)}}(\gamma | l) = \sum_{k=1}^{l} \binom{l}{k} e^{-\frac{k(1-k)\Gamma}{(1-\rho)\Gamma}} e^{-\frac{k\gamma}{(1-\rho)\Gamma}}
\]  
(6)

For given outdated SNR \( \Gamma_{kr_i}^{(o)} \), we obtain the cdf of received SNR at the destination at the \( n \)th ARQ round using the correlation in (2) as
\[
f_{\Gamma_{kr_i}^{(o)}}(\gamma | l) = \frac{e^{-\left(l \gamma / (1-\rho)\Gamma \right)}}{(1-\rho)\Gamma} I_0 \left(\frac{2\sqrt{\rho \gamma \Gamma}}{(1-\rho)\Gamma} \right)
\]  
(7)

where \( I_0(\cdot) \) is the modified zeroth-order Bessel function of the first kind. Multiplying (6) and (7) gives the joint pdf of \( \Gamma_{kr_i}^{(o)} \) and \( \Gamma_{kr_i}^{(o)} \) conditioned on \( [D] \). Then the pdf of \( \Gamma_{kr_i}^{(o)} \) conditioned on \( [D] \) is given by
\[
f_{\Gamma_{kr_i}^{(o)}}(\gamma | D = l) = \frac{\int_{0}^{\infty} f_{\Gamma_{kr_i}^{(o)}}(\gamma | D = l) f_{\Gamma_{kr_i}^{(o)}}(\gamma | D = l) d\gamma}{\int_{0}^{\infty} f_{\Gamma_{kr_i}^{(o)}}(\gamma | D = l) d\gamma}
\]  
(8)

The conditional probability of decoding failure at the destination at \( n \)th ARQ round when \( [D] = l \) is expressed by the pdf of \( \Gamma_{kr_i}^{(o)} \) conditioned on \( [D] \) as
\[
Pr[\Gamma_{kr_i}^{(o)} < \gamma_0 | D = l] = \int_{0}^{\gamma_0} f_{\Gamma_{kr_i}^{(o)}}(\gamma | D = l) d\gamma
\]
where (10) is obtained from the relation
\[
\left( \frac{l}{k} \right) - \left( \frac{l}{k} \right) + \cdots + \left( \frac{l}{k} \right) = 1.
\]

The probability of decoding failure at the destination until the \(N\) th ARQ round is given by
\[
P_{\text{fail}}^{(N)} = \Pr[(\Gamma^{(1)}_{r,d} < \gamma_0) \cap (\Gamma^{(2)}_{r,d} < \gamma_0) \cap \cdots \cap (\Gamma^{(N)}_{r,d} < \gamma_0)].
\]
(11)

As \(\Gamma^{(n)}_{r,d}, n = 1, 2, \cdots, N\), are independent and identically distributed random variables, from (3) and (10), the probability of decoding failure is expressed as
\[
P_{\text{fail}}^{(N)} = \sum_{n=1}^{N} \Pr[\Gamma^{(n)}_{r,d} < \gamma_0]
\]
\[
= \sum_{n=1}^{N} \Pr[D = l] \sum_{k=1}^{N} \Pr[\Gamma^{(n)}_{r,d} < \gamma_0][D = l]
\]
\[
= \sum_{l=1}^{K} \sum_{k=1}^{N} \left( \frac{l}{k} \right) e^{-\frac{k\gamma_0}{\Gamma}} \left( 1 - e^{-\frac{k\gamma_0}{\Gamma}} \right)^{K-l}
\]
\[
= \sum_{k=1}^{K} \sum_{l=1}^{K} \left( \frac{l}{k} \right) e^{-\frac{k\gamma_0}{\Gamma}} \left( 1 - e^{-\frac{k\gamma_0}{\Gamma}} \right)^{K-l}.
\]
(12)

From (4) and (13), the outage probability is given by
\[
P_{\text{out}} = \left( 1 - e^{-\frac{\gamma_0}{\Gamma}} \right)^K + \sum_{l=1}^{K} \left( \frac{K}{l} \right) e^{-\frac{\gamma_0}{\Gamma}} \left( 1 - e^{-\frac{\gamma_0}{\Gamma}} \right)^{K-l}
\]
\[
= \sum_{k=1}^{K} \sum_{l=1}^{K} \left( \frac{l}{k} \right) e^{-\frac{k\gamma_0}{\Gamma}} \left( 1 - e^{-\frac{k\gamma_0}{\Gamma}} \right)^{K-l}.
\]

From (4), (12), (15), and (16), the outage probability is approximated as
\[
P_{\text{out}} \approx \min \left( \frac{\gamma_0}{\Gamma} \right)^K + \sum_{l=1}^{K} \left( \frac{K}{l} \right) \left( \frac{\gamma_0}{\Gamma} \right)^{K-l} N_{(N,l)} 1
\]
\[
\text{where } C(N,I) = \sum_{l=1}^{K} \left( \frac{l}{k} \right) k(1-\rho)^k + \rho
\]
comes from the fact that the outage probability is smaller than one.

IV. NUMERICAL RESULTS

Suppose that \(R = 1 \text{ bps/Hz. The outdated channel by the given correlation } \rho \text{ is simulated according to the algorithm in [8].}

Fig. 2 shows the outage probability of ARQ-based DF opportunistic relaying for \(K = 3, \rho = 0.5\), and various \(N\). It is shown that the simulation results of outage probability match with the analysis and approximation is very close with its analysis at high SNR.

Fig. 3 shows the outage probability of ARQ-based DF opportunistic relaying for \(N = 2, \rho = 0.5\), and various \(K\). It is shown that the analysis match with the simulations for different number of relays.
V. CONCLUSION

In this paper, we analyze the effect of outdated CSI on the outage probability of ARQ-based DF opportunistic relaying. The outage probability is derived in a closed form and it is approximated to a simple form at high SNR. It is shown that the simulation results of outage probability match with its analysis, and the approximation is close to the analysis at high SNR.

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