Performance of Interleaved OFDMA uplink with ICI Self-canceling over Doubly Dispersive Channels

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Abstract-In the interleaved orthogonal frequency-division multiple access (OFDMA) uplink, the received signal from a user is corrupted by the intercarrier interferences (ICI) which result from both his own signal and other users’ signals due to Doppler spreads. Furthermore, different received signal powers and Doppler spreads between users may increase the effect of the ICI. Among the schemes which have been proposed to reduce the ICI in OFDM, the ICI self-canceling scheme is easily applicable to the interleaved OFDMA uplink. In this paper, we investigate the performance of the interleaved OFDMA uplink with and without ICI self-canceling over doubly dispersive channels. We analyzed the signal-to-intercarrier-interference-plus-noise ratio (SINR) improvement and we examined the symbol error rates and the error floors through simulation.

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has emerged as one of the most practical techniques for high-data-rate systems operating over wireless channels and orthogonal frequency-division multiple access (OFDMA) has been proposed for broadband wireless multiple access systems. In OFDMA, the closely spaced subcarriers are assigned to different users for simultaneous transmission. There are two major subcarrier-assignment schemes: subband based one and interleaved one. The interleaved subcarrier-assignment scheme provides maximum frequency diversity and increases the capacity in frequency-selective fading channels. In the interleaved OFDMA system, the orthogonality among subcarriers guarantees that there is no intercarrier interference (ICI), which prevents multiple-access interference (MAI).

OFDMA inherits from OFDM the weakness of being sensitive to any frequency shift in the signal, however, Doppler spread in a doubly dispersive channel destroys the orthogonality among subcarriers and causes the ICI. The ICI decreases the signal-to-intercarrier-interference-plus-noise ratio (SINR) at every subcarrier and results in an error floor. We analyzed the SINR improvement and we examined the symbol error rates and the error floors through simulation.

II. SIR ANALYSIS IN INTERLEAVED OFDMA UPLINK OVER DOUBLY DISPERSE CHANNEL

We consider an interleaved OFDMA system with $K$ users. There are $N$ subcarriers in each OFDM block and all subcarriers are sequentially indexed with $\{i\}, i = 0,1,\ldots, N-1$. $N$ subcarriers are divided into $Q$ subchannels and each subchannel consists of $\lfloor N/Q \rfloor$ subcarriers. The $k$th user is assigned to the $r_k$th subchannel which is a subset of $M$ subcarriers with the index set $\{r_k, Q+r_k,\ldots, (M-1)Q+r_k\}$, $r_k = 0,\ldots,Q-1$. In the discrete time domain, the baseband signal transmitted from the $k$th user can be written as

$$x_n^{(k)} = \frac{1}{\sqrt{N}} \sum_{m=0}^{M-1} S_m^{(k)} e^{j 2\pi (mQ+r_k)}$$

where $S_m^{(k)}$ is the data symbol modulated on the $(mQ+r_k)$-th subcarrier. The superscript $(\cdot)^{\langle k \rangle}$ denotes the $k$th user.

The signals are transmitted through doubly dispersive channels. The complex baseband doubly dispersive channel from the $k$th user can be modeled in the discrete time domain as $h_n^{\langle k \rangle}$, which denotes the tap gain of the $n$th tap at time $n$. The received signal from the $k$th user at time $n$ is corrupted by the ICI resulting from adjacent subcarriers allocated to other users. On the other hand, the ICI-SC is simple and easily applicable to the interleaved OFDMA uplink.

The previous studies [9], [11] presented the analyses of the SINR in the conventional OFDM and the SINR in the ICI self-canceling OFDM over a doubly dispersive channel. In this paper, we analyze the SINR in the interleaved OFDMA uplink with and without ICI self-canceling over doubly dispersive channels and we investigate the performance of the system.

This paper is organized as follows. In Section II, we describe the interleaved OFDMA uplink over doubly dispersive channels and derive the SINR. In Section III, we derive the SINR when the ICI-SC is used in the interleaved OFDMA uplink. Then, in Section IV, we present some numerical and simulation results. Finally, conclusions are drawn in Section V.
We assume the typical WSSUS model [10], under which different delays are uncorrelated
\[
E\left[ h_{m}^{(k)}(t) h_{m}^{(k)}(t - \Delta t) \right] = r_{m}^{(k)}(\Delta t) \sigma_{m}^{(k)} \delta(\Delta t)
\]  
(2)
where \( r_{m}^{(k)}(\cdot) \) denotes the normalized tap autocorrelation, \( \sigma_{m}^{(k)} \) represents the normalized variance of the \( k \)th tap such that \( \sum_{\nu = 0}^{L-1} \sigma_{m}^{(k)}(\nu)^{2} = 1 \), and \( \delta(\cdot) \) is Kronecker delta function.

At the uplink receiver, the signal of one OFDMA block is the superposition of signals from all \( K \) users. Assuming all \( K \) users are synchronized in time, the received baseband signal at time \( n \) is given by
\[
y_{n} = \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} \sqrt{P_{\text{in}}} h_{m}^{(k)}(t) x_{m,n}^{(k)} + w_{n}^{(k)}
\]  
(3)
where \( P_{\text{in}}^{(k)} \) is the average received power of the \( k \)th user’s signal at the base station and \( w_{n}^{(k)} \) is the additive white Gaussian noise with variance \( \sigma_{n}^{2} \). From the FFT of \( \{y_{n}\} \), the demodulated signal on the \( nQ + r_{k} \)-th subcarrier is obtained and it can be expressed as
\[
Z_{m}^{(k)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_{n} e^{-j2\pi\left(mQn + r_{k}\right)/N}
\]  
(4)
\[
= \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} \sqrt{P_{\text{in}}} \sum_{t=0}^{L-1} h_{m}^{(k)}(t) e^{-j2\pi(mQn + r_{k})/N}
\]  
(5)
}\[
= \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} \sqrt{P_{\text{in}}} \sum_{t=0}^{L-1} h_{m}^{(k)}(t) e^{-j2\pi(mQn + r_{k})/N} + W_{m}^{(k)}
\]  
(6)
where \( W_{m}^{(k)} = \sum_{n=0}^{N-1} w_{n} e^{-j2\pi(mQn + r_{k})/N} / \sqrt{N} \). For simplicity, let us define \( H_{m}^{(k)}(n) = \sum_{t=0}^{L-1} h_{m}^{(k)}(t) e^{-j2\pi(mQn + r_{k})/N} \). Then (6) can be rewritten as
\[
Z_{m}^{(k)} = \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} \sqrt{P_{\text{in}}} \sum_{t=0}^{L-1} H_{m}^{(k)}(n) e^{-j2\pi(mQn + r_{k})/N}
\]  
(7)
\[
G_{m}^{(k)}(p) = \frac{1}{N} \sum_{n=0}^{N-1} H_{m}^{(k)}(n) e^{-j2\pi pm/N}
\]  
(8)
which is the Doppler-variant transfer function at the \( nQ + r_{k} \)-th subcarrier, and
\[
S_{m}^{(k)}(\phi) = \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} r_{m}^{(k)(\phi)} S_{m}^{(k)} + W_{m}^{(k)}
\]  
(9)
\[
S_{m}^{(k)}(\phi) = \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} r_{m}^{(k)(\phi)} S_{m}^{(k)} + W_{m}^{(k)}
\]  
(10)
where \( S_{m}^{(k)}(\phi) \) denotes the Doppler spectrum of the channel from the \( k \)th user.

Using (7) and (10), we can express the SINR at the \( nQ + r_{k} \)-th subcarrier as (11) at the bottom of the page.

III. SIR ANALYSIS IN INTERLEAVED OFDMA UPLINK WITH ICI-SC OVER DOUBLY DISPERSIVE CHANNEL

In the ICI self-canceling scheme, each data symbol is modulated on \( B \) adjacent subcarriers, with weights equal to the coefficients of the polynomial \( (1 - D)^{B} \), where \( D \) denotes one subcarrier delay in the frequency domain. In this paper, we consider the specific case of \( B = 2 \). For using the ICI self-canceling scheme in the interleaved OFDMA uplink, the index set of the \( r_{k} \)th subchannel is modified and equals to \( \{2r_{k}, 2r_{k} + 1, 2Q + 2r_{k}, 2Q + 2r_{k} + 1, \ldots, (2M-1)Q + 2r_{k}, 2r_{k}, 2r_{k} + 1, 2Q + 2r_{k}, 2Q + 2r_{k} + 1, \ldots, (2M-1)Q + 2r_{k}\} \).
2(\bar{M} - 1)Q + 2r_1 + 1} \), where \( \bar{M} = M / 2 \). The modulated symbols on the \((2mQ + 2r_1)\)-th and the \((2mQ + 2r_1 + 1)\)-th subcarrier are given by \( S_m^{(e)} \) and \(-S_m^{(e)} \) respectively. Following a similar set of steps in the section II, the received signals at the \((2mQ + 2r_1)\)-th and the \((2mQ + 2r_1 + 1)\)-th subcarrier are given by

\[
Z_m^{(e)}(m) = \sum_{k=1}^{M-1} \sum_{n=0}^{Q-1} \sqrt{P_m^{(e)}} \left[ G_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) - G_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2 - 1) \right] S_m^{(e)} + W_m^{(e)}
\]

and

\[
Z_m^{(e)}(m) = \sum_{k=1}^{M-1} \sum_{n=0}^{Q-1} \sqrt{P_m^{(e)}} \left[ G_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) - G_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2 - 1) \right] (-S_m^{(e)}) + W_m^{(e)}
\]

where \( G_m^{(e)}(p) \) is the Doppler-variant transfer function of the \( k \)-th user at the \((2mQ + 2r_1 + 1)\)-th subcarrier. After the ICI canceling demodulation [1], we get the resultant signal

\[
\tilde{Z}_m^{(e)} = Z_m^{(e)} - Z_m^{(e)}(m)
\]

\[
= \sum_{k=1}^{M-1} \sum_{n=0}^{Q-1} \sqrt{P_m^{(e)}} G_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) S_m^{(e)} + W_m^{(e)}
\]

where

\[
\tilde{G}_m^{(e)}(p) = -G_m^{(e)}(p - 1) + G_m^{(e)}(p) + G_m^{(e)}(p) - G_m^{(e)}(p + 1).
\]

Using (19) and (20), we can express the SINR of the \( m \)-th symbol for the \( k \)-th user as (21) at the bottom of the page. It is derived in the appendix A that

\[
E \left\{ \left| \tilde{G}_m^{(e)}(p) \right|^2 \right\}
\]

can be expressed as

\[
E \left\{ \left| \tilde{G}_m^{(e)}(p) \right|^2 \right\} = \left( S^{(e)}(\phi) * \tilde{V}_{ISC}^{(e)}(\phi) \right)_{\omega = 2\pi f_p / N}
\]

where \( \tilde{V}_{ISC}^{(e)}(\phi) \) is given by (23) at the bottom of the page and \( \omega = \sin^2(\pi l / N) \). (See the appendix B)

Note that (21) is similar to (20) except that \( E \left\{ \left| \tilde{G}_m^{(e)}(p) \right|^2 \right\} \) replaces \( E \left\{ \tilde{G}_m^{(e)}(p) \right\} \). Therefore, we can infer that \( \tilde{V}_{ISC}^{(e)}(\phi) \) reflects the effect of the ICI self-canceling scheme.

IV. NUMERICAL AND SIMULATION RESULTS

In this section, we present some numerical and simulation results. We consider an interleaved OFDMA uplink system with \( N = 512 \) and four users. Each user is allocated \( M = 128 \) subcarriers. We use the two-path Rayleigh fading channel with equal gains. Unless otherwise mentioned, we assume that the Doppler frequencies of the users are same and the ratio between the average received powers at the base station is given by \( P^{(1)} : P^{(2)} : P^{(3)} : P^{(4)} = 1 : 2 : 1 : 2 \).

A. Numerical Results

Fig. 1 presents the SINR improvement when using the ICI-SC in the interleaved OFDMA uplink. We use the classical Doppler spectrum (Jakes’ model)

\[
P_j(f) = \begin{cases} 
\frac{1}{\pi \sqrt{f_d^2 - f^2}}, & \text{if } |f| < f_d \\
0, & \text{else}
\end{cases}
\]

and we substitute \( S^{(e)}(\phi) = P_j(\phi / N / 2\pi f_d) \) into (12) and (22), where \( T_d \) is the duration of an OFDM block. In Fig. 1, the signals with strong received power have about 5dB higher SIR than those with weak received power in the system without the ICI-SC. The ICI-SC improves all the users’ SIRs. For example, about 15dB performance improvement is achieved at the normalized Doppler frequency \( f_d T_d = 0.1 \). As the normalized Doppler frequency increases, the SIR improvement decreases. Note that the SIR improvement of the signals with strong received power decreases relatively slowly.

\[
\bar{S}INR_m^{(e)} = \frac{\sum_{k=1}^{\bar{M}-1} \sum_{n=0}^{Q-1} P_m^{(e)} \left| \tilde{G}_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) \right|^2}{\sigma_m^2}
\]

\[
\bar{V}_{ISC}^{(e)}(\phi) = \frac{4\pi^2 \sin^2(\phi N / 2) N^6}{\phi^2 - \pi / N \phi^2 + \pi / N \phi^2} + \frac{4\pi^2 \sin^2(\phi N / 2) N^4}{\phi^2 - \pi / N \phi^2 + \pi / N \phi^2} \sum_{l=0}^{\bar{M}-1} \omega_l \left( \sigma_l^{(e)} \right)^2
\]

\[
\sum_{k=1}^{\bar{M}-1} \sum_{n=0}^{Q-1} P_m^{(e)} \left| \tilde{G}_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) \right|^2 - P^{(e)} \left| \tilde{G}_m^{(e)}(0) \right|^2 + \sigma_m^2
\]

\[
\sum_{k=1}^{\bar{M}-1} \sum_{n=0}^{Q-1} P_m^{(e)} \left| \tilde{G}_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) \right|^2 - P^{(e)} \left| \tilde{G}_m^{(e)}(0) \right|^2 + \sigma_m^2
\]

\[
\sum_{k=1}^{\bar{M}-1} \sum_{n=0}^{Q-1} P_m^{(e)} \left| \tilde{G}_m^{(e)}(2Q(m' - m) + 2r_1 - 2r_2) \right|^2 - P^{(e)} \left| \tilde{G}_m^{(e)}(0) \right|^2 + \sigma_m^2
\]
quickly.

B. Simulation Results

We used quaternary phase-shift keying (QPSK) with coherent demodulation in simulation. Fig. 2 shows the error floors versus the normalized Doppler frequency. The ICI-SC lowers the error floors since it improves all the users' SIRs. Fig. 3 and Fig. 4 show the SER versus SNR of each user. In Fig. 3, we assume that all the users have the same Doppler spreads $f_d T_s = 0.1$. The SERs of the users are different because the signals of the users have the different received powers and experience the different ICI power. The ICI-SC improves the SER performance but the SER difference between the users with different received powers still remains.

In Fig. 4, we assume that all the users have equal power but they have different Doppler spreads: $f_d T_s = [0.05, 0.05, 0.3, 0.05]$. In the system without the ICI-SC, ‘user 3’ has the best performance because the other users have the relatively lower Doppler spreads and the signal of ‘user 3’ is less corrupted by the intercarrier interference. However, note that ‘user 1’ has the best performance when the ICI-SC is used. This is because the Doppler spread of ‘user 1’ is smaller than that of ‘user 3’ and he has the relatively larger signal energy after ICI canceling demodulation. Therefore, ‘user 1’ has the larger SINR.

V. CONCLUSIONS

In this paper, we investigated the interleaved OFDMA uplink over doubly dispersive channels. Simulation results show that the SER of a user is considerably affected by the received powers and the Doppler spreads of the other users. We employed the ICI-SC to solve this problem of the interleaved subcarrier-assignment scheme which provides maximum frequency diversity. The SIR improvement is calculated by the numerical analysis. Our numerical and simulation results show that the ICI-SC improves the performance of the interleaved OFDMA uplink over doubly dispersive channels.
APPENDIX A

Following a similar set of steps taken in [4], we will derive the variance of \( \hat{G}_m^{(\nu)} (p) \) below. Let us define
\[
\hat{G}_k (p) = \frac{1}{N} \sum_{n=0}^{N-1} \hat{H}_k (n) e^{-j2\pi np/N} \tag{A.1}
\]
where
\[
\hat{H}_k (n) = \sum_{l=0}^{L-1} h_{n,l} e^{-j2\pi kl/N} \tag{A.2}
\]
Note that \( k \) is the subcarrier index here and \( u \) is the user index.

Let us define
\[
\bar{G}_k (p) = -\hat{G}_{k+1} (p-1) + \hat{G}_k (p) + \hat{G}_{k+1} (p) - \hat{G}_k (p+1) \tag{A.3}
\]
Then, we can see that \( \hat{G}_m^{(\nu)} (p) = \bar{G}_{2mQ+2\nu} (p) \) and \( \hat{G}_m^{(\nu)} (p) = \bar{G}_{2mQ+2\nu} (p) \). Now we will derive the variance of \( \hat{G}_k (p) \). For convenience, we omitted the superscript \( (\nu) \) for the user index.

The variance is expanded as
\[
E \left[ \left( \bar{G}_k (p) \right) \right] = E \left[ \left( \hat{G}_{k+1} (p-1) \right) \right] + E \left[ \left( \hat{G}_k (p) \right) \right] + E \left[ \left( \hat{G}_{k+1} (p+1) \right) \right]
\]
\[
-2 \text{Re} \left[ E \left[ \left( \hat{G}_k (p) \hat{G}_{k+1} (p-1) \right) \right] - E \left[ \left( \hat{G}_k (p) \hat{G}_{k+1} (p) \right) \right] \right]
\]
\[
+2 \text{Re} \left[ E \left[ \left( \hat{G}_k (p+1) \hat{G}_{k+1} (p-1) \right) \right] - E \left[ \left( \hat{G}_k (p+1) \hat{G}_{k+1} (p) \right) \right] \right]
\]
\[
+2 \text{Re} \left[ E \left[ \left( \hat{G}_k (p+1) \hat{G}_{k+1} (p) \right) \right] + E \left[ \left( \hat{G}_k (p) \hat{G}_{k+1} (p+1) \right) \right] \right] \tag{A.4}
\]

From (5) and (6), we obtain
\[
E \left[ \left( \bar{G}_{k+1} (p + p_1) \hat{G}_{k+1} (p + p_2) \right) \right] = \frac{1}{N^2} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \sum_{l=0}^{L-1} \sum_{l=0}^{L-1} E \left\{ h_{n_1,l} h_{n_2,l} \right\}
\]
\[
\times e^{-j2\pi (l_1-k_1)/N} e^{-j2\pi (l_2-k_2)/N} e^{-j2\pi (p_1n_1-p_2n_2)/N} \times e^{-j2\pi (p_1n_1-p_2n_2)/N} \times e^{-j2\pi (p_1n_1-p_2n_2)/N} \tag{A.5}
\]
To decompose (A.5) into the Doppler spectrum term and the other term, we will use the approach similar to that developed in the ICI-generating mechanism [4]. (A.5) can be rewritten as
\[
E \left[ \left( \bar{G}_{k+1} (p + p_1) \hat{G}_{k+1} (p + p_2) \right) \right] = \frac{1}{N^2} \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi (l_1-k_1)/N} e^{-j2\pi (l_2-k_2)/N} e^{-j2\pi (p_1n_1-p_2n_2)/N} \times \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} r(n_1-n_2) e^{-j2\pi (p_1n_1-p_2n_2)/N} \times \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} e^{-j2\pi (p_1n_1-p_2n_2)/N}
\]
\[
= \frac{1}{N^2} \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi (l_1-k_1)/N} e^{-j2\pi (l_2-k_2)/N} \tag{A.6}
\]
where \( \sigma_l^2 \) is the variance of the Doppler spectrum and \( \sigma_l^2 \) denotes the Doppler spectrum and \( \sigma_l^2 \) is the variance of \( \hat{G}_k (p) \) for convenience, we omitted the superscript \( (\nu) \) for the user index.

Using \( V^{(\nu)} (q) \), we can write
\[
E \left[ \left( \bar{G}_{k+1} (p + p_1) \right) \hat{G}_{k+1} (p + p_2) \right] = \frac{1}{N^2} \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi (l_1-k_1)/N} \times \sum_{q=-\infty}^{\infty} v(q) r(q) e^{-j2\pi (p_1n_1-p_2n_2)/N} \tag{A.7}
\]
In (A.7), \( S (\phi) \) denotes the Doppler spectrum and \( V^{(\nu)} (\phi) \) is the DTFT of \( N^{-2} V^{(\nu)} (q) \).

\[
V^{(\nu)} (\phi) = \frac{1}{N^2} \sum_{q=-\infty}^{N-1} v^{(\nu)} (q) e^{-j\phi q}, \quad \phi \in \mathbb{R} \tag{A.8}
\]
After some manipulation, we get
\[
V^{(\nu)} (\phi) = \frac{\sin^2 (\phi N/2)}{N^2 \sin^2 (\phi/2) \sin (\phi/2 - \pi \Delta p/ N) / N} \tag{A.9}
\]
With the definition of \( V^{(p,p_2)} (\phi) \) such as
\[
V^{(p_1,p_2)} (\phi) \triangleq V^{(\nu)} (\phi + 2\pi p_1 / N) \times \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi l/N} \tag{A.10}
\]
we can express (A.7) more simply such as
\[
E \left[ \left( \bar{G}_{k+1} (p + p_1) \right) \hat{G}_{k+1} (p + p_2) \right] = \left[ S (\phi) \ast V^{(p_1,p_2)} (\phi) \right] \bigg|_{q=2\pi p_2/N} \tag{A.10}
\]
Applying (A.10) to (A.4), we have

\[
E \left[ \left\| \tilde{G}_1 (\rho) \right\| \right] = \left[ S (\phi) * V_{\text{ISC}} (\phi) \right]_{\rho = 2 \pi / N} \tag{A.11}
\]

where

\[
V_{\text{ISC}} (\phi) = V_{0}^{(-1,-1)} (\phi) + 2 \Im V_{0}^{(0,0)} (\phi) + V_{0}^{(1,1)} (\phi)
\]

\[
-2 \operatorname{Re} \left\{ V_{-1}^{(0,-1)} (\phi) - V_{0}^{(0,0)} (\phi) + V_{0}^{(1,0)} (\phi) \right\}
\]

\[
-2 \operatorname{Re} \left\{ -V_{-1}^{(0,1)} (\phi) + V_{0}^{(1,0)} (\phi) + V_{0}^{(0,1)} (\phi) \right\} \tag{A.12}
\]

**APPENDIX B**

Here, we will derive the approximate of \( V_{\text{ISC}} (\phi) \).

\[
\operatorname{Re} \left\{ V_{-1}^{(0,-1)} (\phi) + V_{0}^{(1,0)} (\phi) \right\} = \frac{1}{N^2} \sin^2 (\phi N / 2) \left[ 2 \cos \left( \frac{\pi (l + 1)}{N} \right) \cos \left( \frac{\pi l}{N} \right) \right]
\]

\[
\times \sum_{l=0}^{L-1} \sigma_l^2 \tag{B.1}
\]

\[
\operatorname{Re} \left\{ V_{-1}^{(1,0)} (\phi) + V_{0}^{(0,1)} (\phi) \right\} = \frac{1}{N^2} \sin^2 (\phi N / 2) \left[ 2 \cos \left( \frac{\pi (l + 1)}{N} \right) \cos \left( \frac{\pi l}{N} \right) \right]
\]

\[
\times \sum_{l=0}^{L-1} \sigma_l^2 \tag{B.2}
\]

\[
\operatorname{Re} \left\{ V_{-1}^{(1,-1)} (\phi) \right\} = \frac{1}{N^2} \sin^2 (\phi N / 2) \left[ 2 \cos \left( \frac{\pi (l + 1)}{N} \right) - 1 \right]
\]

\[
\times \sum_{l=0}^{L-1} \sigma_l^2 \tag{B.3}
\]

If we assume that \( \phi < 1 \) and the number of subcarriers \( N \) is large, we have the following approximations:

\[
\cos \left( \frac{\pi (l + 1)}{N} \right) \approx \cos \left( \frac{\pi l}{N} \right) \tag{B.4}
\]

and

\[
\sin \left( \frac{\phi + \pi}{2} - \frac{\pi}{N} \right) = \frac{\phi + \pi}{2} - \frac{\pi}{N} \tag{B.5}
\]

Using the above approximations and the property of the trigonometric functions, (B.1-B.3) simplify to

\[
\operatorname{Re} \left\{ V_{-1}^{(0,-1)} (\phi) + V_{0}^{(1,0)} (\phi) \right\} = \frac{1}{N^2} \sin^2 \left( \phi N / 2 \right) \left[ \sum_{l=0}^{L-1} \sigma_l^2 \sin^2 \left( \frac{\pi l}{N} \right) \right]
\]

\[
\times \sum_{l=0}^{L-1} \sigma_l^2 \cdot 2 \left( 1 - \sin^2 \left( \frac{\pi l}{N} \right) \right) \tag{B.6}
\]

Substituting (B.6-B.8) into (A.12) and using (B.4-B.5), we can obtain

\[
V_{\text{ISC}} (\phi) = \frac{4 \pi^4 \sin^2 \left( \phi N / 2 \right) / N^4}{\left( \phi / 2 - \pi / N \right)^2 \left( \phi / 2 + \pi / N \right)^2 \sum_{l=0}^{L-1} \sigma_l^2} + \frac{4 \pi^2 \sin^2 \left( \phi N / 2 \right) / N^4}{\left( \phi / 2 - \pi / N \right) \left( \phi / 2 + \pi / N \right)^2 \sum_{l=0}^{L-1} \alpha_l \sigma_l^2} \tag{B.9}
\]

where \( \alpha_l = \sin^2 \left( \frac{\pi l}{N} \right) \).

**REFERENCES**


