Frequency-Domain Partial Response Coding for Alamouti SFBC-OFDM System in Doubly Selective Channels

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SUMMARY Time variation within an OFDM symbol causes inter-carrier interference (ICI). In this letter, frequency-domain partial response coding (PRC) is investigated to reduce ICI in the Alamouti SFBC-OFDM system. Based on the expression of the ICI power in the SFBC-OFDM system with PRC, the near-optimal weights of PRC are derived. Simulation results show that the PRC scheme can reduce ICI effectively.

key words: inter-carrier interference (ICI), SFBC-OFDM, partial response coding (PRC)

1. Introduction

OFDM is robust against frequency-selective fading but sensitive to time-selective fading. Time variation within an OFDM symbol destroys orthogonality among subcarriers and causes inter-carrier interference (ICI), which results in an error floor if not compensated for. To reduce the effect of ICI in the OFDM system with a single transmit antenna, coding techniques, such as ICI self-cancellation [1] and frequency-domain partial response coding (PRC) [2], have been studied.

It is well known that antenna diversity is effective in reducing the degrading effects of a fading channel. And space-frequency coding (SFC) is a means of combining the advantages of transmit antenna diversity and OFDM. In the absence of ICI, the space-frequency block coded (SFC) OFDM system has much better symbol error rate (SER) performance than the OFDM system with a single transmit antenna. However, ICI reduces the advantage of antenna diversity gain in the SFBC-OFDM system and also results in error floor.

In this letter, we applied frequency-domain PRC to the Alamouti SFBC-OFDM system, in order to reduce the effect of ICI. In the SFBC-OFDM system with PRC, we derive the expression of the ICI power caused by Doppler frequency shift and obtain the near-optimal PRC weights which minimize ICI. It is shown that the proposed PRC weights reduce ICI while wrong selected PRC weights increase ICI.

2. System Model

Figure 1 shows the baseband model of the Alamouti SFBC-OFDM system with PRC. PRC encoded sequences can be represented as

\[ s_p = \sum_{i=0}^{K-1} c_i x_{p-i} \]  

(1)

where \( x_p \)'s are the symbols to be transmitted, \( E[|x_p|^2] = 1 \), and \( c_i \)'s are PRC weights with unit norm, i.e., \( \sum_{i=0}^{K-1} c_i^2 = 1 \). The sequences \( \{s_p\} \) are encoded in space and frequency by the Alamouti code and the transmitted signals in time domain can be expressed as

\[ y^{(1)}(t) = \sum_m (s_{2m} e^{j2\pi f_s t} + s_{2m+1} e^{j2\pi f_{s+1} t}) \]  

(2)

\[ y^{(2)}(t) = \sum_m (-s_{2m} e^{j2\pi f_{s+1} t} + s_{2m+1} e^{j2\pi f_s t}) \]  

(3)

where \( f_p = f_0 + p\Delta f \) is the frequency of the \( p \)th subcarrier, \( \Delta f = 1/T_s \) is the subcarrier spacing, and \( T_s \) is one OFDM symbol duration. Here, the superscript \( (i) \) indicates the \( i \)th transmit antenna and \( (\cdot)^* \) denotes the complex conjugate.

As stated in [2], PRC weights for frequency-flat fading channels are applicable to the frequency-selective fading channel since path delays are usually much smaller than OFDM symbol duration. Therefore, we will assume a frequency-flat fading channel for the convenience of mathematical tractability in deriving meaningful PRC weights.

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The received signal in a frequency-flat and time-selective fading channel is given by
\[ z(t) = y^{(1)}(t)g^{(1)}(t) + y^{(2)}(t)g^{(2)}(t) \]  
where \( y^{(i)}(t) \) is the channel impulse response from the \( i \)th transmit antenna to the receive antenna. We assume that \( y^{(1)}(t) \) and \( y^{(2)}(t) \) are independent and have the same statistics. In addition, we assume \( E\left[ y^{(i)}(t)^2 \right] = 0.5, i = 1, 2 \).

The output of the FFT at the \( p \)th subcarrier becomes
\[ r_p = \frac{1}{T_s} \int_0^{T_s} z(t)e^{-j2\pi f_p t} \, dt. \]  
After some manipulation, the outputs can be expressed as
\[ r_{2m} = a_0 s_{2m} - b_0 s_{2m+1} + f_{2m}^{(1)} + f_{2m}^{(2)} \]  
\[ r_{2m+1} = a_0 s_{2m+1} + b_0 s_{2m} + f_{2m+1}^{(1)} + f_{2m+1}^{(2)} \]  
where \( a_0 \) and \( b_0 \) are the gains of the desired signals and \( f_{2m}^{(i)} \) denotes the sum of the interfering signals originating from the \( i \)th transmit antenna. Here, \( a_q \) and \( b_q \) are defined respectively as
\[ a_q = \frac{1}{T_s} \int_0^{T_s} y^{(1)}(t)e^{-j2\pi q f_p t} \, dt, \]  
\[ b_q = \frac{1}{T_s} \int_0^{T_s} y^{(2)}(t)e^{-j2\pi q f_p t} \, dt. \]  
And \( f_{2m}^{(i)} \)'s can be written as follows:
\[ f_{2m}^{(1)} = \sum_{n=0} a_{2n} s_{2m-2n} + \sum_{n=0} a_{2n-1} s_{2m-2n-1}, \]  
\[ f_{2m}^{(2)} = -\sum_{n=0} b_{2n} s_{2m-2n} + \sum_{n=0} b_{2n-1} s_{2m-2n-1}, \]  
\[ f_{2m+1}^{(1)} = \sum_{n=0} a_{2n+1} s_{2m-2n} + \sum_{n=0} a_{2n} s_{2m-2n-1}, \]  
\[ f_{2m+1}^{(2)} = -\sum_{n=0} b_{2n+1} s_{2m-2n} + \sum_{n=0} b_{2n} s_{2m-2n-1}. \]

As shown in Fig. 1, \( \tilde{s}_p \) can be obtained from \( r_p \)'s by the simple Alamouti decoding and \( \bar{s}_p \) can be recovered by a maximum-likelihood sequence detector (MLSD).

In the absence of ICI, \( \bar{s}_{2m} \) and \( \bar{s}_{2m+1} \) are obtained respectively as
\[ \bar{s}_{2m} = a_0 r_{2m} + b_0 r_{2m+1} = (|a_0|^2 + |b_0|^2) s_{2m}, \]  
\[ \bar{s}_{2m+1} = a_0 r_{2m+1} - b_0 r_{2m} = (|a_0|^2 + |b_0|^2) s_{2m+1}. \]  
(14) and (15) indicates that the Alamouti SFBC-OFDM system achieves the diversity order of two. Since the diversity order affects the slope of the SER, the Alamouti SFBC-OFDM system has much better SER performance than the OFDM system with a single transmit antenna if ICI is suppressed.

3. PRC Weights for Alamouti SFBC-OFDM

The total ICI power at the \( p \)th subcarrier is defined as
\[ P_{ICI,p} = E\left[ |f_{p}^{(1)}|^2 + |f_{p}^{(2)}|^2 \right]. \]  
Based on the central limit theorem, \( f_{p}^{(i)} \) can be modeled as a zero mean Gaussian random process [3]. Since \( y^{(1)}(t) \) and \( y^{(2)}(t) \) are independent, \( f_{p}^{(1)} \) and \( f_{p}^{(2)} \) are independent and uncorrelated, i.e., \( E\left[ f_{p}^{(1)} f_{p}^{(2)} \right] = 0 \). Then, (16) reduces to
\[ P_{ICI,p} = E\left[ |f_{p}^{(1)}|^2 + |f_{p}^{(2)}|^2 \right]. \]  
It is derived in Appendix A that the total ICI power \( P_{ICI,p} \) can be derived as
\[ P_{ICI,p} = 1 - 2 \int_0^{f^2} P(f) \sin^2 (fT_s) \, df + \bar{T}_{PRC,p} (c_K, f_s T_s) \]  
where \( \bar{T}_{PRC,p} (c_K, f_s T_s) \) is given by
\[ \bar{T}_{PRC,p} (c_K, f_s T_s) = \int_0^{f^2} \frac{\sin^2 (\pi f T_s) P(f)}{\pi^2} \, df \]  
and \( \bar{T}_{PRC,p} (c_K, f_s T_s) \) is given by (20), shown at the top of the next page. Here \( P(f) \) is the power spectral density of \( y^{(i)}(t) \), \( i = 1, 2 \), and \( f_s \) is the maximum Doppler frequency shift.

To reduce the ICI power, we need to find the optimum \( c_K \) which minimizes \( \bar{T}_{PRC,p} (c_K, f_s T_s) \). Since \( f^2 T_s \gg 1 \) for \( 0 \leq f \leq f_s \), \( \bar{T}_{PRC,2m} (c_K, f_s T_s) \) and \( \bar{T}_{PRC,2m+1} (c_K, f_s T_s) \) can be approximated respectively as \( g_s (c_K) \) and \( g_o (c_K) \), which are given (21) and (22).

It is derived in Appendix B that \( g_s (c_K) = g_o (c_K) = c_K^H R_K c_K \) where \( R_K = S_K^H + Q_K^H + Q_K \) and \( Q_K = [Q_{ij}]_{K \times K} \) are defined as
\[ S_{ij} = \begin{cases} 4/(2k)^2, & \text{if } j = i + 2k \text{ and } k \neq 0 \\ 0, & \text{otherwise} \end{cases} \]  
and
\[ Q_{ij} = \begin{cases} 8k/[2(2k - 1)(2k + 1)^2], & \text{if } j = i + 2k - 1 \\ 0, & \text{otherwise}. \end{cases} \]

\( R_K \) is real symmetric and Hermitian since \( S_K \) is a symmetric matrix. Using the theorem about Rayleigh-Ritz ratio [5, Theorem 4.2.2], we have
\[ \lambda_{\min} \leq c_K^H R_K c_K \leq \lambda_{\max} \]  
where \( \lambda_{\min} \) and \( \lambda_{\max} \) are the smallest and largest eigenvalues of \( R_K \). The left equality in (23) holds if the \( c_K \) is the normalized eigenvector of \( R_K \) corresponding to the eigenvalue \( \lambda_{\min} \).

\( R_K \) is not positive semidefinite because all the diagonal entries of \( R_K \) is zero and \( R_K \) is not diagonally dominant [6, Theorem 4.2.6]. There must be a negative eigenvalue \[5, \text{Theorem 7.2.1}] and \( \lambda_{\min} \) is negative. This means that the optimum PRC weight vector makes \( \bar{T}_{PRC,p} (c_K, f_s T_s) \) negative and reduces the total ICI power \( P_{ICI,p} \) in (16) while wrong selected PRC weight vector may increase the total ICI power.
from 5 weights of type I improve the WER performances in the figure, we can see that the PRC weights of type II deteriorate the OFDM system with a single transmit antenna. From the proposed PRC weights introduced in [2], which are optimized for the OFDM system with and without PRC. Two types of PRC varying fading channel is generated by Jakes’ model. A two-path Rayleigh fading channel model is used and the time-symbol forms an RS codeword. In our simulation, a two-frequency-selective and time-selective fading channel and is divided into 128 subchannels. 120 tones at the middle are used to transmit data. QPSK with coherent demodulation is employed. The (40,30) Reed-Solomon (RS) code is utilized to correct the burst errors resulting from frequency-selective fading. Each code symbol in the RS code consists of three QPSK symbols grouped in frequency. Hence, each OFDM symbol forms an RS codeword. In our simulation, a two-path Rayleigh fading channel model is used and the time-varying fading channel is generated by Jakes’ model.

Figure 2 shows the WER performances of the SFBC-OFDM system with and without PRC. Two types of PRC weights are shown in Table 1. Type I denotes the set of the proposed PRC weights and type II represents the set of the PRC weights introduced in [2], which are optimized for the OFDM system with a single transmit antenna. From the figure, we can see that the PRC weights of type II deteriorate the WER performances. However, the proposed PRC weights of type I improve the WER performances in the high SNR region. Furthermore, the error floor is reduced from 5.6×10⁻³ to 1.3×10⁻³ by using the 4-tap PRC weights.

5. Conclusions

In this letter, we have applied frequency-domain PRC to the Alamouti SFBC-OFDM system. Under the assumption of a frequency-flat and time-selective fading channel, the near-optimal PRC weights are derived to minimize ICI. Simulation results in the two-path Rayleigh fading channel show that the proposed PRC weights are also applicable to the frequency-selective and time-selective fading channel and improve the WER performance in the high SNR region.

4. Simulation Results

In this section, we provide our simulation results for word error rate (WER) performance of the PRC scheme in the SFBC-OFDM system with two transmit antennas and one receive antenna. The entire channel bandwidth, 1.25 MHz, is divided into 128 subchannels. 120 tones at the middle are used to transmit data. QPSK with coherent demodulation is employed. The (40,30) Reed-Solomon (RS) code is utilized to correct the burst errors resulting from frequency-selective fading. Each code symbol in the RS code consists of three QPSK symbols grouped in frequency. Hence, each OFDM symbol forms an RS codeword. In our simulation, a two-path Rayleigh fading channel model is used and the time-varying fading channel is generated by Jakes’ model.

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### References


Appendix A

Here we explain the derivation process of $P_{ICL,p}$. Substituting (1) into (11) and using $E(x_m,c_m^*) = \delta_{mn}$, where $\delta_{mn}$ is Kronecker delta function, we have

$$E[f_{2m}^2] = E \left\{ \left| \sum_{n \neq 0} \sum_{i=0}^{K-1} b_{2n}c_i x_{2m-2n+1-i}^2 + \sum_{n \neq 0} \sum_{i=0}^{K-1} b_{2n}c_i x_{2m-2n-i}^2 \right|^2 \right\}$$

where

$$V_{2m}^{(2)} = \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ \sum_{n_1}^{U(i)} \sum_{k=Kc(i)} E(b_{2n}b_{2n-2k}^*) - E(b_{2k}b_{2k}^*) \right] c_i c_{i+2k}$$

$$- \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ \sum_{n_1}^{U(i)} \sum_{k=Kc(i)} E(b_{2n}b_{2n-2k+1}^*) - E(b_{2k}b_{2k+1}^*) \right] c_i c_{i+2k-1}$$

$$- \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ \sum_{n_1}^{U(i)} \sum_{k=Kc(i)} E(b_{2n-1}b_{2n-2k}^*) - E(b_{2k-1}b_{2k+1}^*) \right] c_i c_{i+2k-1}$$

$$+ \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ \sum_{n_1}^{U(i)} \sum_{k=Kc(i)} E(b_{2n-1}b_{2n-2k-1}^*) - E(b_{2k-1}b_{2k+1}^*) \right] c_i c_{i+2k}$$

with $U(i) = [(K-1-i)/2], U_2(i) = [(K-i)/2], L(i) = [-i/2], and L_2(i) = [(i-1)/2].$

It is derived in [4] that

$$E[|a_n|^2] = E[|b_n|^2] = \frac{1}{2} \int_0^{\pi} P(f) \sin^2(fT_s) df$$

And the autocorrelation of $a_i$ or $b_i$ is shown in [2] that

$$E(a_m a_n^*) = E(b_m b_n^*) = \int_{-1}^{+1} (T_s z) \frac{\sin \pi |m-n||z|}{\pi |m-n|} e^{-j \pi (m-n)z} dz$$

(A-1)

when $m \neq n$. Here, $R(\tau)$ is the autocorrelation of $\gamma(t)$. From (A-1) and $\sum_{m=-\infty}^{\infty} e^{-j 2\pi (2x)} = \delta(2x - m)$ [2, Appendix], we have

$$V_{2m}^{(2)} = \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ - E(b_{2n}b_{2k}^*) - E(b_{2k}b_{2n}^*) \right] c_i c_{i+2k}$$

$$\sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ - E(b_{2n}b_{2k+1}^*) - E(b_{2k+1}b_{2n}^*) \right] c_i c_{i+2k-1}$$

$$- \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \left[ - E(b_{2n-1}b_{2k+1}^*) - E(b_{2k+1}b_{2n-1}^*) \right] c_i c_{i+2k-1}$$

(A-2)

since

$$\sum_{n_1}^{U(i)} \sum_{k=Kc(i)}^{U(i)} E(b_{2n}b_{2n-2k}) = \sum_{n_1}^{U(i)} \sum_{k=Kc(i)}^{U(i)} E(b_{2n}b_{2n-2k+1}) = 0$$

Substituting (A-1) and $R(\tau) = \int_0^{\pi} P(f) \cos(2\pi f\tau) df$ into (A-2), we can obtain

$$V_{2m}^{(2)} = \int_0^{\pi} \frac{\sin^2(\pi fT_s)P(f)}{\pi} \left[ \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \frac{2}{(2k)^2 - f^2 T_s^2} c_i c_{i+2k}$$

$$\sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \frac{1}{(2k+1)^2 - f^2 T_s^2} (c_i c_{i+2k-1} + c_{i+1} c_{i+2k-1}) \right] df.$$  

In the same way, we have

$$E[f_{2m}^2] = \sum_{n \neq 0} E[|a_n|^2] + V_{2m}^{(1)}$$

where

$$V_{2m}^{(1)} = \int_0^{\pi} \frac{\sin^2(\pi fT_s)P(f)}{\pi} \left[ \sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \frac{2}{(2k)^2 - f^2 T_s^2} c_i c_{i+2k}$$

$$\sum_{i=0}^{K-1} \sum_{k=Kc(i)}^{U(i)} \frac{1}{(2k+1)^2 - f^2 T_s^2} (c_i c_{i+2k-1} + c_{i+1} c_{i+2k-1}) \right] df.$$  

From the above results and (17), the total ICI power $P_{ICL,2m}$ can be expressed as

$$P_{ICL,2m} = 1 - 2 \int_0^{\pi} P(f) \sin^2(fT_s) df$$

$$+ \tilde{I}_{PRC,2m} (\epsilon_K, f_d T_s)$$

(A-3)

where $\tilde{I}_{PRC,2m} (\epsilon_K, f_d T_s)$ is given by

$$\tilde{I}_{PRC,2m} (\epsilon_K, f_d T_s)$$
\[
= \int_0^{T_s} \frac{\sin^2(\pi f T_s) P(f)}{\pi^2} I_{PRC_{2m}}(e_K, f T_s) df \quad (A.4)
\]

with
\[
\tilde{I}_{PRC_{2m}}(e_K, f T_s) = \sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \frac{4}{(2k)^2 - f^2 T_s^2} c_i c_{i+2k} + \sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \left( \frac{1}{(2k - 1)^2 - f^2 T_s^2} - \frac{1}{(2k + 1)^2 - f^2 T_s^2} \right)
\times (c_i c_{i+2k-1} + c_i c_{i+2k-1}). \quad (A.5)
\]

In the same way, we can derive \( P_{IC_{1,2m+1}} \).

**Appendix B**

Here we derive that \( g_o(e_K) = g_o(e_K) = c_K^H R_K e_K \). The sums in (21) and (22) can be expressed as the products of the vectors and matrices as follows:

\[
\sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \frac{4}{(2k)^2} c_i c_{i+2k} = \sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} c_i S_{ij} c_i = c_K^H S_K^T c_K,
\]

\[
\sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \frac{8k}{(2k - 1)^2 (2k + 1)^2} c_i c_{i+2k-1} = \sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \frac{8k}{(2k - 1)^2 (2k + 1)^2} c_i c_{i+2k-1}.
\]

\[
= \sum_{j=0}^{K-1} \sum_{i=0}^{K-1} c_i^T Q_j c_i = c_K^H Q_K^T c_K,
\]

\[
= \sum_{i=0}^{K-1} \sum_{k=0}^{L(i)} \frac{8k}{(2k - 1)^2 (2k + 1)^2} c_i^T c_{i+2k-1}
\]

\[
= \sum_{i=0}^{K-1} c_i^T Q_j c_j = c_K^H Q_K c_K.
\]

Therefore, we can obtain

\[
g_o(e_K) = c_K^H S_K^T e_K + c_K^H Q_K^T e_K + c_K^H Q_K c_K
\]

\[
= c_K^H (S_K^T + Q_K^T + Q_K) e_K
\]

\[
= c_K^H R_K e_K. \quad (A.6)
\]

If we define \( Q'_{K} = \left[ \begin{array}{c} \mathbf{Q}'_k \\ K \end{array} \right] \) as

\[
Q'_j = \begin{cases} 
-8(k-1) & \text{if } j = i + 2k - 1 \\
0 & \text{otherwise}
\end{cases}
\]

then we have \( Q'_K = Q_K^T \). In the same way, we can easily get \( g_o(e_K) = c_K^H R_K e_K \).