Hybrid ARQ Scheme Using Interleaved Reed–Solomon Codes in a Power-Controlled DS-CDMA Cellular System

Chaehag Yi, *Member, IEEE,* and Jae Hong Lee, *Member, IEEE*

***Abstract*—In this paper, a new hybrid automatic repeat request (ARQ) scheme is proposed for data transmission in a power-controlled direct sequence (DS) code division multiple access (CDMA) system cellular system. Data frame is composed of interleaved Reed–Solomon codes. The depth of interleaving is determined by a power-control interval. After decoding each codeword with algebraic decoding, the post-decoding processor decides whether to accept the codeword or to discard it by using channel state information from the power-control processor. The proposed hybrid ARQ scheme significantly reduces the probability of undetected error among accepted codewords without significantly reducing throughput.**

I. Introduction

A DIRECT sequence (DS) code division multiple access (CDMA) system increases channel capacity in a mobile communication system [1]. In a DS-CDMA system, many users transmit signals over the same frequency spectrum simultaneously using unique pseudonoise (PN) sequences. Since PN sequences are not ideally orthogonal on a reverse link, interferences between users arise that cause a near–far problem. To overcome the near–far problem, feedback power control is used to enable a base station to receive equal power from each user. For a fading channel, power commands are issued at a higher rate than the fading rate to compensate fast fading [2].

In a coded system for a fading channel, interleaving and soft decision decoding are used to improve its performance [3]. Interleaving converts burst errors into random errors. Soft decision decoding algorithms, such as generalized minimum distance decoding algorithm and Chase algorithm, are used for block codes [4], [5]. Soft decision requires channel state information (CSI) for each code symbol. If decoding failure is allowed in a decoding algorithm, the algorithm achieves lower probability of undetected error than without decoding failure [6].

In this paper, a new hybrid automatic repeat request (ARQ) scheme is proposed for data transmission in a power controlled DS-CDMA cellular system. In the scheme, an error detecting algorithm uses CSI for each code symbol, which is obtained from feedback power control.

In Section II, a new hybrid ARQ scheme is proposed. In Section III, simulation results for its performance are given. The conclusion is given in Section IV.

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II. System Model

A. Hybrid ARQ Scheme in a Power-Controlled CDMA Cellular System

A hybrid ARQ scheme is applied to a reverse link of a power controlled DS-CDMA cellular system. An M-ary orthogonal signal set is used for modulation. The size of an orthogonal signal set M is selected to be the same as the alphabet size q of a Reed–Solomon (RS) code with ith component denoted by c_i, $i = 1, 2, \cdots, n$.

The block diagram of a receiver is shown in Fig. 1. It employs two branch antenna diversity to combat the effects of multipath. Not knowing the phase of a received signal due to fading in the channel, the demodulator detects a received signal noncoherently. Each correlator in a correlator bank is matched to one of the M orthogonal signals.

With the outputs z_k^i, $k = 1, 2, \cdots, M$, from the square-law combiner, the decision device selects the largest to give an estimated code symbol r_i, $i = 1, 2, \cdots, n$. For an orthogonal signal set, the largest output of the square-law combiner is proportional to signal power and the rest of outputs are proportional to noise and interference power. Power control interval T_p is selected so that its normalized value by the fading rate f_D is less than 0.1. Suppose that there are m channel symbols in T_p. The channel state information (CSI) for an estimated code symbol r_i is given by

$$\lambda_i = \frac{\sum_{j=1}^{m} \max\left(z_{j,1}^i \cdots z_{j,M}^i\right)}{\sum_{j=1}^{m} \sum_{k=1,\, k\neq \max}^{m} z_{j,k}^i} \tag{1}$$

where $z_{j,k}^i$ is the output of the kth square-law combiner at the jth symbol in ith power control period. In (1) the numerator and the denominator are average signal power and interference component during m channel symbols, respectively. Let $\hat{y}_i$ denote the estimated signal to interference ratio (SIR) for an ith power control period. The values of $\hat{y}$ corresponding to the

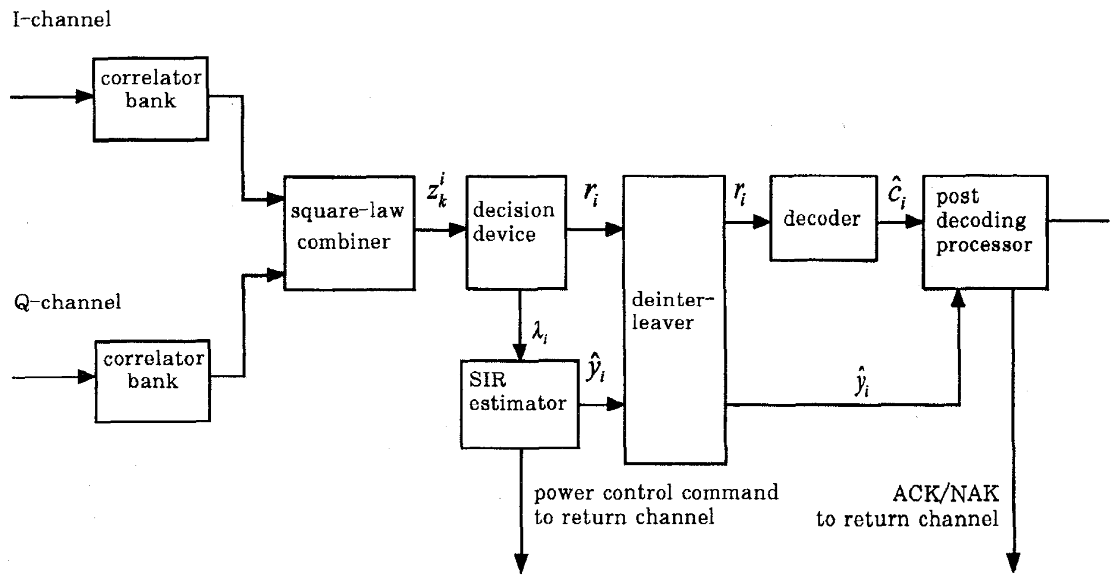

Fig. 1. Receiver block diagram.

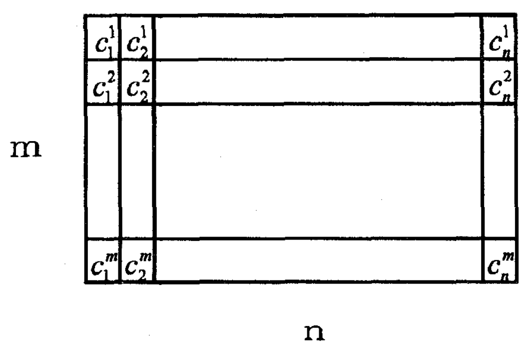

Fig. 2. Data frame.

various values of λ are obtained from computer simulation and stored in a lookup table. The SIR estimator gives an estimated SIR $\hat{y}_i$ from λ_i for each symbol by using the lookup table [2]. The estimated code symbols and corresponding SIR's are deinterleaved. From a decoded codeword and estimated SIR's for each code symbol, the post-decoding processor decides whether to accept the codeword or to discard it.

A data frame for a hybrid ARQ is shown in Fig. 2. It is an $m \times n$ array composed of interleaved RS codewords with a code symbol being an element of the array c_i^j, $j = 1, 2, \cdots, m$, $i = 1, 2, \cdots, n$. Interleaving depth m and interleaving length n are set to be the same as the power control interval and the codelength, respectively. Each codeword in a data frame is individually accepted or rejected for retransmission.

B. Error-Detecting Algorithm

The result of bounded distance decoding is categorized into three types: correct decoding, decoding failure, and decoding error. Decoding failure occurs when a received word is not contained in a decoding sphere of any codeword. Decoding failure is a kind of error detection.

We propose a new decoding algorithm that improves the performance of an RS code with little increase in complexity. With the hard-quantized symbol r_i, error/erasure algebraic decoding is performed. The decoder outputs the decoded codeword $\hat{c}$ unless decoding failure happens. Decoding failure is regarded as error detection. The post decoding processor performs error detection with $\hat{c}$ and the estimated SIR $\hat{y}_i$.

With noncoherent detection of an M-ary orthogonal signal, the error probability of a code symbol c_i with estimated SIR $\hat{y}_i$ is given by

$$P_i^s = \sum_{l=1}^{M-1} \frac{(-1)^{l+1}}{l+1} \binom{M-1}{l} \exp\left(-\frac{l}{l+1}\hat{y}_i\right). \tag{2}$$

For convenience, the order of components in a codeword is rearranged so that it becomes $\hat{y}_1 \leq \hat{y}_2 \leq \cdots \leq \hat{y}_n$. The symbols of estimated SIR $\hat{y}_i$ below a preset erasure threshold Y_s are erased. Let P_{er} denote the probability of symbol erasure.

Now, error detection is performed. The reliability of a codeword $\hat{c}$ computed by the post decoding processor is given by

$$R(\hat{c}) = \prod_{i \in F} P_{er} \prod_{i \in E} P_i^s \prod_{i \notin (E \cup F)} (1 - P_i^s) \tag{3}$$

where F is the set of erased components with $|F| = f$ and E the set of components estimated as errors with $|E| = b$ and $b \leq (d_{\min} - f - 1)/2$. $R(\hat{c})$ the probability that the codeword $\hat{c}$ was sent. Let A denote a set of $d_{\min} - f - b$ least reliable components other than components in E among non-erased components. Each element of A is contained in $\{f + 1, \cdots, d_{\min}\}$. The set A determines a codeword $\acute{c}$ that is $d_{\min}$ apart from $\hat{c}$. The two codewords $\hat{c}$ and $\acute{c}$ have different components in set A, E, and F. The codeword $\acute{c}$ does exist from the following property: for given any set of $d_{\min}$ components, a maximum distance separable code like an RS code contains $q-1$ codewords of weight $d_{\min}$ of which only nonzero components are at the given $d_{\min}$ components [7, p. 309]. The symbol value of $\acute{c}$, however, is not obtained from set A. Let a relative threshold for error detecting be defined as a probability that a received vector has error in A, which is given by

$$T_R = \prod_{i\in F} P_{er} \prod_{i\in A} P_i^s \prod_{i\notin(A\cup F)} (1-P_i^s). \quad (4)$$

T_R is the probability that codeword $\acute{c}$ was sent. In other words, T_R is the reliability of codeword $\acute{c}$. It is known from the definition of A that the codeword $\acute{c}$ has the maximum reliability among the codewords apart $d_{\min}$ from $\hat{c}$ since the components of two codewords $\hat{c}$ and $\acute{c}$ are different in the possible least reliable components including erasures. If $R(\hat{c})$ is greater than T_R, then $\hat{c}$ has the maximum reliability among the codewords, apart from $d_{\min}$itself. In this case, the post-decoding processor accepts $\hat{c}$. If T_R is greater than $R(\hat{c})$, $\hat{c}$ has the smaller reliability than that of $\acute{c}$. In this case, the post-decoding processor discards $\hat{c}$ since there exists another codeword that has higher reliability than that of $\hat{c}$ obtained from algebraic decoding.

From (3) and (4), the reliability test becomes

$$\prod_{i\in F} P_{er} \prod_{i\in E} P_i^s \prod_{i\notin(E\cup F)} (1-P_i^s) \lesseqgtr \prod_{i\in F} P_{er} \prod_{i\in A} P_i^s \prod_{i\notin(A\cup F)} (1-P_i^s). \quad (5)$$

Taking the logarithm on both sides after simplification, (5) becomes

$$\sum_{i\in E} \ln \frac{P_i^s}{1-P_i^s} \lesseqgtr \sum_{i\in A} \ln \frac{P_i^s}{1-P_i^s}. \quad (6)$$

To reduce the time for reliability test, precomputed values of $\ln[P_i^s/(1-P_i^s)]$ are stored in a lookup table of the post-decoding processor.

Let P_{UE} denote the probability of undetected decoding error and P_{DF} denote the probability of decoding failure. Both P_{UE} and P_{DF} are obtained as a function of a preset threshold bit energy to interference ratio E_b/I_0 and parameters of power control from computer simulation. Let P_E denote the probability of undetected decoding error among accepted codewords, in short, the probability of decoding error. It is given by

$$P_E = \frac{P_{UE}}{1-P_{DF}}. \quad (7)$$

Each codeword in a data frame is individually accepted or rejected for retransmission. Suppose that selective repeat retransmission protocol is used. The normalized throughput of ARQ scheme is given by

$$\eta = \frac{k}{n}(1-P_{DF}) \quad (8)$$

where k/n is the code rate of an RS code.

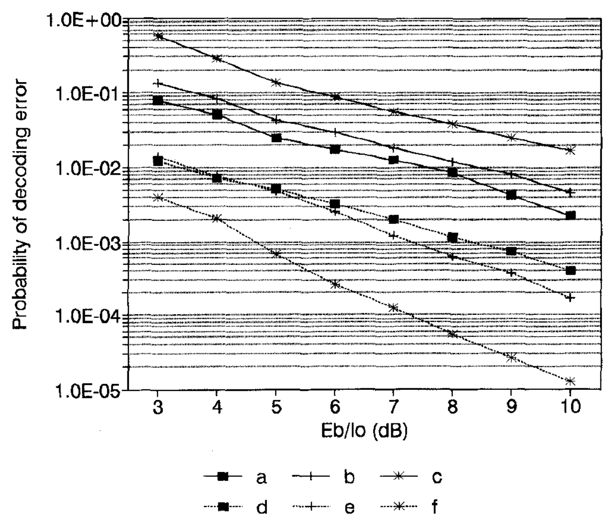

Fig. 3. Probability of undetected error among accepted codewords of RS (7, 3) code for symbol erasure threshold. $f_D T_P = 0.01$, $M = 8$. (a) $Y_s = 3.5$ dB, (b) $Y_s = 5.5$ dB, (c) $Y_s = 7.5$ dB, (d) $Y_s = 3.5$ dB with EDA, (e) $Y_s = 5.5$ dB with EDA, and (f) $Y_s = 7.5$ dB with EDA.

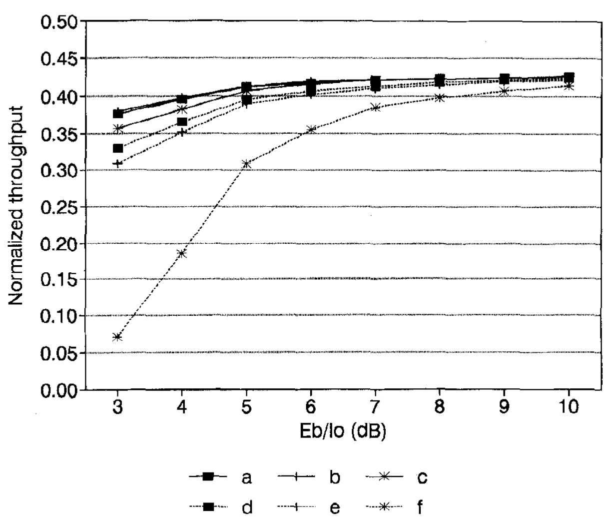

Fig. 4. Normalized throughput of RS (7, 3) code for symbol erasure threshold. $f_D T_P = 0.01$, $M = 8$. (a) $Y_s = 3.5$ dB, (b) $Y_s = 5.5$ dB, (c) $Y_s = 7.5$ dB, (d) $Y_s = 3.5$ dB with EDA, (e) $Y_s = 5.5$ dB with EDA, and (f) $Y_s = 7.5$ dB with EDA.

III. Simulation Results

The performance of the proposed hybrid ARQ scheme is evaluated for a power-controlled DS-CDMA cellular system by computer simulation. Suppose that transmitter power is updated by 0.5 dB every power control interval and that peak transmitter power is limited by 3 dB above a preset bit energy

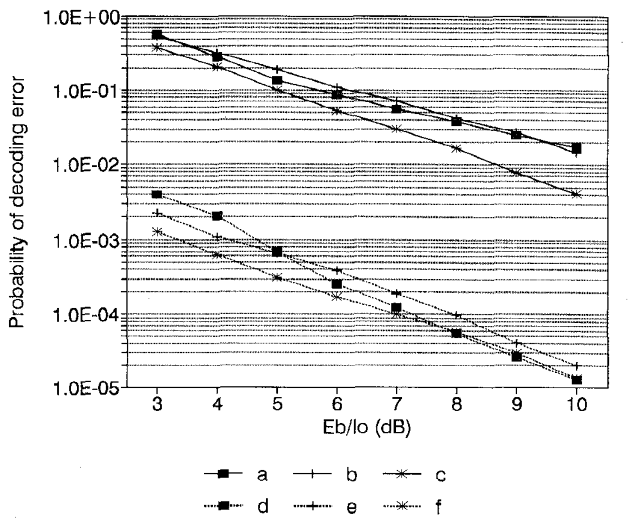

Fig. 5. Probability of undetected error among accepted codewords of RS (7, 3) code for normalized power control interval. $M = 8$, $Y_s = 7.5$ dB. (a) $f_DT_P = 0.01$, (b) $f_DT_P = 0.02$, (c) $f_DT_P = 0.05$, (d) $f_DT_P = 0.01$ with EDA, (e) $f_DT_P = 0.02$ with EDA, and (f) $f_DT_P = 0.05$ with EDA.

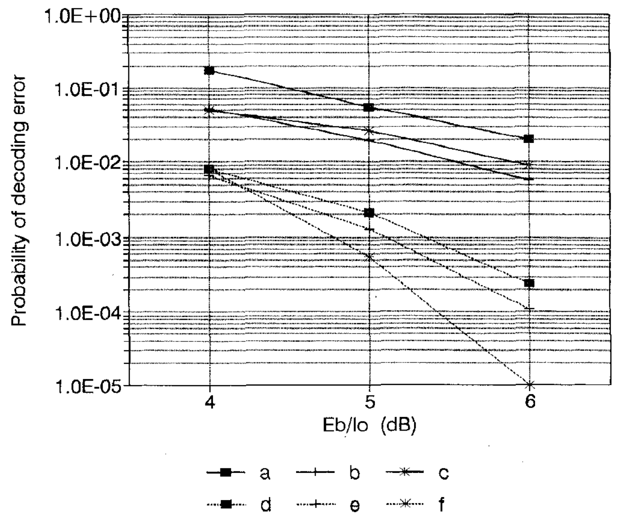

Fig. 7. Probability of undetected error among accepted codewords for alphabet size. $f_DT_P = 0.01$, $Y_s = 7.5$ dB. (a) RS (15, 11), (b) RS (31, 23), (c) RS (63, 47), (d) RS (15, 11) with EDA, (e) RS (31, 23) with EDA, and (f) RS (63, 47) with EDA.

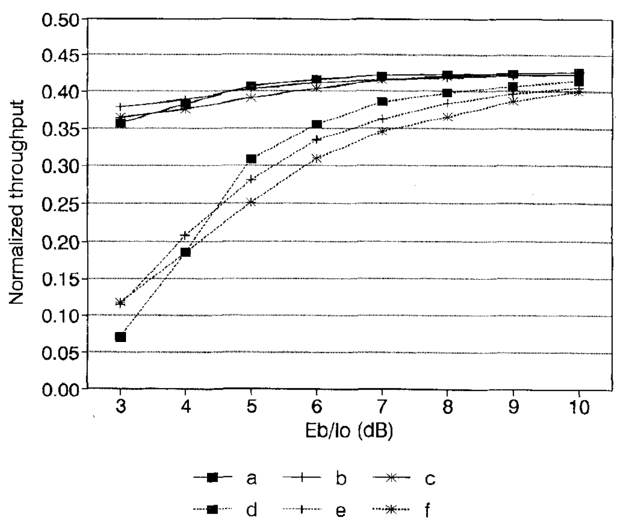

Fig. 6. Normalized throughput of RS (7, 3) code for normalized power control interval. $M = 8$, $Y_s = 7.5$ dB. (a) $f_DT_P = 0.01$, (b) $f_DT_P = 0.02$, (c) $f_DT_P = 0.05$, (d) $f_DT_P = 0.01$ with EDA, (e) $f_DT_P = 0.02$ with EDA, and (f) $f_DT_P = 0.05$ with EDA.

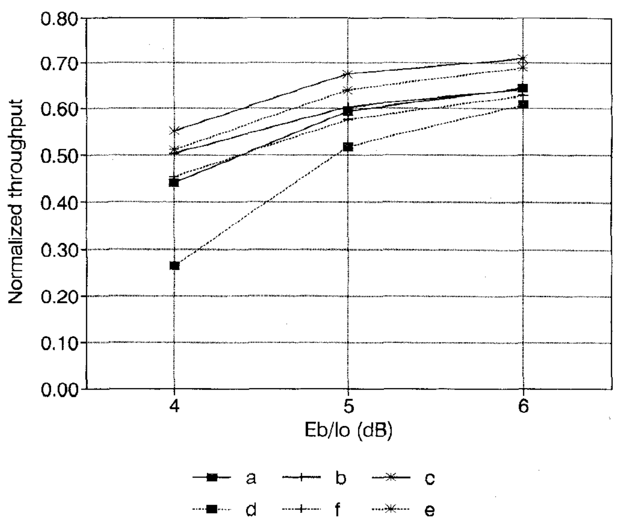

Fig. 8. Normalized throughput for alphabet size. $f_DT_P = 0.01$, $Y_s = 7.5$ dB. (a) RS (15, 11), (b) RS (31, 23), (c) RS (63, 47), (d) RS (15, 11) with EDA, (e) RS (31, 23) with EDA, and (f) RS (63, 47) with EDA.

to interference ratio (E_b/I_0) threshold. With this assumption, the average bit energy to interference ratio is almost equal to the preset E_b/I_0 threshold. Also suppose that a return channel for power control commands has the bit error probability of 0.1. We approximate other user interference as a Gaussian variable with zero mean. From (7) and (8) the probability of decoding error P_E and normalized throughput are obtained as a function of a preset E_b/I_0 threshold and parameters of power control.

In Fig. 3, the probability of decoding error P_E is plotted for various values of symbol erasure threshold Y_s with RS (7, 3) code, $f_DT_P = 0.01$, and $M = 8$. It is shown that P_E is much smaller with the error-detecting algorithm (EDA) than without EDA. P_E decrease as Y_s increases with EDA, while P_E increases as Y_s increases without EDA. In Fig. 4, normalized throughput is plotted for Y_s with RS (7, 3) code, $f_DT_P = 0.01$, and $M = 8$. It is shown that η is minimally affected by EDA for small Y_s and large E_b/I_0.

In Fig. 5, P_E is plotted for various normalized power control interval f_DT_P with RS (7, 3) code, $M = 8$, and $Y_s = 7.5$ dB. It is shown that P_E is not significantly affected by f_DT_P when f_DT_P is less than 0.1. In Fig. 6, η is plotted for various f_DT_P with RS (7, 3) code, $M = 8$, and $Y_s = 7.5$ dB. It is shown that η is not much affected by f_DT_P when f_DT_P is less than 0.1.

In Fig. 7, P_E is plotted for a few codes with similar code rate but different alphabet size $q(= M)$ with $f_DT_P = 0.01$,

$Y_s = 7.5\,\text{dB}$. It is shown that P_E is significantly reduced by using EDA for large q. In Fig. 8, η is plotted for a few codes with similar code rate but different alphabet size q with $f_D T_P = 0.01$, $Y_s = 7.5\,\text{dB}$. It is shown that η is more influenced by code rate than by EDA.

IV. Conclusion

We propose a hybrid automatic repeat request (ARQ) scheme for data transmission in a power-controlled DS-CDMA cellular system. This hybrid ARQ scheme can be easily applied to any data frame. Feedback power control provides channel state information for each code symbol. The channel state information for each code symbol is utilized in an error-detecting algorithm to decide whether to accept the estimated codeword or to discard it. The error detecting algorithm significantly reduces the probability of undetected error among accepted codewords with little throughput reduction.

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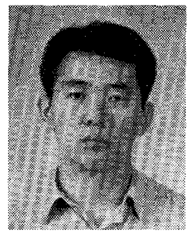

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