A Two-Stage Adaptive MMSE Detector for Synchronous DS-CDMA System in a Rayleigh Fading Channel

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Abstract—In this paper, a two-stage adaptive minimum mean-squared error (MMSE) detector for synchronous DS-CDMA system is considered in a Rayleigh fading channel. The near-far effect causes performance degradation of a DS-CDMA system. To improve performance of the adaptive MMSE detector in a near-far environment, a near-far resistant decorrelating decision-feedback detector (DDFD) is combined with the adaptive MMSE detector in which a normalized least mean square (NLMS) algorithm is utilized as an adaptive algorithm. The performance of the proposed receiver is evaluated by simulation and compared with that of other suboptimum multiuser detectors. Through the numerical results, it is shown that the proposed receiver achieves significant performance improvement at the expense of a modest increase in computational complexity.

I. INTRODUCTION

A direct-sequence code-division multiple-access (DS-CDMA) system suffers from MAI and near-far effect. Multiuser detectors are adopted to mitigate these performance degradation factors. Suboptimum multiuser detectors are proposed to reduce computational complexity with marginal performance degradation compared with the optimum multiuser detector which achieves low error probability at the expense of high computational complexity [1], [2]. As a suboptimum multiuser detector, the adaptive MMSE detector achieves better performance than the matched filter (MF) receiver without much increase of computational complexity [2], [6]. In a near-far environment, however, the performance of the adaptive MMSE detector is degraded by MAI from stronger user signals in spite of its MAI cancellation function. In most previous works, either power control or a detector employing successive interference cancellation (SIC) is used to mitigate the near-far effect. The power control may increase a receiver’s complexity and induce severe near-far effect due to estimation error of received signal power. The detector employing SIC such as a decorrelating decision-feedback detector (DDFD) utilizes bit decisions of stronger user signals and achieves near-far resistance although there is an error propagation problem [3], [4].

In this paper, a two-stage adaptive MMSE detector is proposed for synchronous DS-CDMA system in a Rayleigh fading channel. In the proposed receiver, the DDFD is combined with the adaptive MMSE detector. After the MAI is regenerated using the DDFD output and subtracted from the received signal, the remaining interference is suppressed in the adaptive MMSE detector. The bit error rate (BER) of the proposed receiver is evaluated by computer simulation and compared with that of other suboptimum multiuser detectors. The rest of this paper is organized as follows. In section II, the system model is described. In section III, a two-stage adaptive MMSE detector is proposed. Simulation results are presented in section IV and conclusions are drawn in section V.

II. SYSTEM MODEL

Consider a synchronous DS-CDMA system with \( K \) simultaneous users. A binary data of each user is multiplied by a unique signature sequence to produce a transmitted baseband signal. The signature sequence for the \( k \) th user is given by

\[
a_k(t) = \sum_{n=0}^{N-1} a_k[n] p(t-nT_c)
\]

for \( 0 \leq t \leq T \) (1)

where \( N \) is a processing gain, \( a_k[n] \) is the \( n \) th element of the signature sequence, \( T_c \) is a chip duration, \( T = NT_c \) is a bit duration, and \( p(t) \) is a unit amplitude normalized rectangular chip-pulse with duration \( T_c \). The \( m \) th transmitted baseband signal for the \( k \) th user is given by

\[
s_k(t) = P_r b_k[m] a_k(t-mT)
\]

for \( 1 \leq k \leq K \) (2)

where \( P_r \) is a transmit power with a constant value for all users and \( b_k[m] \in \{1,-1\} \) is the \( m \) th BPSK modulated information bit.

The \( m \) th transmitted signal experiences a Rayleigh fading with a near-far effect. The impulse response of a Rayleigh fading channel \( a_k(t) \) is given by

\[
a_k(t) = \sum_{l=1}^{L} c_{kj} \delta(t - \tau_{kj})
\]

where \( L \) is the number of paths, \( c_{kj} \) and \( \tau_{kj} \) are the fading gain and propagation delay of the \( j \) th path for the \( k \)th
received baseband signal for the \( k \) th user is given by

\[
x_k(t) = \sqrt{P} b_k[m] \sum_{j=1}^{M} c_{kj}(t-mT-r_{kj})
\]

where \( \sqrt{P} \) is the received signal amplitude. It is assumed that a Rayleigh fading channel has a single-path for all users, i.e., \( L = 1 \), the fading gain \( c_{kj} \) is constant for a bit duration, and the arrival time delay for each user is equal, i.e., \( r_{11} = r_{21} = \cdots = r_{k1} = 0 \). It is also assumed that the received signal amplitude decreases as the user index increases. The \( m \) th baseband signal received at the receiver is given by

\[
r(t) = \sum_{j=1}^{K} x_j(t) + n(t)
\]

\[
= \sum_{j=1}^{K} A_j b_j[m] a_j(t-mT) + n(t)
\]

where \( A_j \) is the received signal amplitude for the \( j \) th user and \( n(t) \) is a white Gaussian noise with two-sided power spectral density of \( \sigma^2 = N_0/2 \) W/Hz.

### III. A TWO-STAGE ADAPTIVE MMSE DETECTOR

The proposed receiver is a form of the adaptive MMSE detector combined with DDFD for synchronous DS-CDMA system. At the first stage, the MAI is regenerated using the bit estimates and estimated received amplitudes obtained from the DDFD and decorrelator, respectively. After the regenerated MAI is subtracted from the received signal, at the second stage, the remaining interference is suppressed using NLMS algorithm in the adaptive MMSE algorithm.

The received signal is despreaded by the chip-matched filter bank. The chip-matched filter bank output vector for the \( m \) th bit of all users is given by

\[
y = [y_1, y_2, \ldots, y_K]^T = RA_b + \hat{n}
\]

where \( R \) is the \((K \times K)\) cross-correlation matrix of signature sequences for all users, \( A = \text{diag}[A_1, A_2, \ldots, A_K] \) is the received signal amplitude matrix, \( b \) is the vector of the \( m \) th transmitted bits, and \( \hat{n} = [\hat{n}_1, \hat{n}_2, \ldots, \hat{n}_K]^T \) is a white Gaussian noise vector with the auto-correlation \( \sigma^2 R \). The \( k \) th element of \( y \) is given by

\[
y_k = A_k b_k[m] + \sum_{j=1, j \neq k}^{K} A_j b_j[m] r_{kj} + \hat{n}_k
\]

where \( r_{kj} \) is the cross-correlation between the \( j \) th and \( k \) th signature sequences.

The despreaded signals by MF bank are the input to the DDFD and decorrelator. In the DDFD, the bit estimates of MAI corresponding to stronger users are reliable, the possibility of doubling the MAI value will be doubled. However, since the DDFD outputs of \( A_1 \) and \( A_2 \) are subtracted from the received signal, at the first stage, the MAI is regenerated using the bit estimates from the DDFD, the estimated received signal amplitudes, and signature sequences, it is subtracted from the received signal. The regenerated MAI is given by

\[
\hat{I}(t) = \sum_{j=1, j \neq k}^{K} \hat{A}_j b_j[m] a_j(t-mT) + n(t)
\]

where \( \hat{A}_j \) is the estimated received signal amplitude and \( \text{sgn}(\cdot) \) is given by

\[
\text{sgn}(n) = \begin{cases} 1, & n \geq 0 \\ -1, & n < 0 \end{cases}
\]

The \( \hat{A}_j \) is obtained by the amplitude estimator using the MF bank output, given by

\[
\hat{A}_j = (R^{-1}y) \left| \begin{array}{c} \hat{A}_j \end{array} \right| = (Ab + R^{-1}\hat{n}) \left| \begin{array}{c} \hat{A}_j \end{array} \right|
\]

where \( | \cdot | \) represents an absolute value and \( R^{-1} \) is an inversion matrix of \( R \).

If the \( i \) th user is the desired user, all other \((K-1)\) user signals are considered as MAI. After the MAI is regenerated by multiplying the bit estimates from the DDFD, the estimated received signal amplitudes, and signature sequences, it is subtracted from the received signal. The regenerated MAI is given by

\[
I(t) = \sum_{j=1, j \neq k}^{K} \hat{A}_j b_j[m] a_j(t-mT) + n(t)
\]

The regenerated MAI is subtracted from the received signal, given by

\[
r(t) = r(t) - \hat{I}(t)
\]

\[
= A_K b_K[t] - \hat{A}_j b_j[m] a_j(t-mT) + n(t)
\]

If the bit estimates of MAI from the DDFD are correct, only the desired user signal and a white Gaussian noise remain in (14). Conversely, if the bit estimates are incorrect, the MAI value will be doubled. However, since the DDFD outputs of stronger user signals are reliable, the possibility of doubling the MAI corresponding to stronger users is relatively lower than that of weaker users. The bit estimate of the desired user is obtained from the adaptive MMSE detector. The adaptive MMSE detector is an \( N \)-tap adaptive FIR filter to minimize the mean squared error (MSE) between the transmitted bit and filter output [5], [6]. A number of adaptive algorithms based on the MMSE criterion are available, ranging from the very fast recursive least square (RLS) to somewhat slower but very simple least mean square (LMS) algorithms. In this paper, the NLMS algorithm is adopted to obtain the filter coefficient vector.

\[
z_k = F_k A_k b_k[m] + \sum_{j=1}^{K-1} F_k A_j b_j[m] + n_k
\]
The $m$th bit estimate of the desired user is given by
\[ \hat{b}_m = \text{sgn}(\varepsilon_m) = \text{sgn}(r^T w_n) \] (15)
where $r_n^* = [r_{1,n}, r_{2,n}, \ldots, r_{N,n}]^T$ is the vector representation of $r(t)$ and $w_n = w + h_n = [w_{1,n}, w_{2,n}, \ldots, w_{N,n}]^T$ is the filter coefficient vector for the desired user and an adaptive component vector. The adaptive component vector is updated by the NLMS algorithm
\[ h_n = [h_{a,1}, h_{a,2}, \ldots, h_{a,N}]^T \]
\[ = h_{a,n-1} + 2\mu_n (\hat{b}_n - v_{a,n}) r_n^* \]
where $\mu_n$ is the time-variant step size which controls the rate of convergence of the algorithm, and $e_n$ is an error component with a reference variable $\hat{b}_n$ obtained by a decision-directed mode. The step size is expressed as $\mu_n = \mu_0 r_n^* e_n^*$ with a fixed value $\mu$ between 0 and 1.

IV. NUMERICAL RESULTS

Consider a chip and symbol synchronous DS-CDMA system in a Rayleigh fading channel with a near-far effect. Suppose that the number of users is 10 and the signature sequence for each user is randomly selected among the Gold sequence of length 31. The generator polynomials of signature sequence are given by $g_1(D) = 1 + D^2 + D^3$ and $g_2(D) = 1 + D^2 + D^3 + D^4 + D^5$. Assume that the channel for each user is a frequency-flat Rayleigh fading channel based on the Jakes' model with normalized fading rates $f_d T = 0.01$ and 0.001 where $f_d$ is the maximum Doppler frequency [8]. Since it is assumed a channel with a near-far effect, the received power difference between user signals is $10 \log (A_d/A_w) = 1 \text{ dB}$. Also assume that the receiver has perfect synchronization and knows the signature sequences of all users. The adaptive algorithm used in simulations is the NLMS algorithm with $\mu_n = \{1/(100 - N) \} r_n^* e_n^*$ [9].

Fig. 1 shows the BER for the weakest user of proposed receiver and compares those of other suboptimum multiuser detectors: the MF receiver, the adaptive MMSE detector, and the DDFD. Since, in a near-far environment, the performance for the weakest user is generally inferior to that for other users, the BER for the weakest user is evaluated. Fig. 1(a) and Fig. 1(b) show the BER in a fast ($f_d T = 0.001$) and a slow ($f_d T = 0.001$) Rayleigh fading channel, respectively. It is shown in both figures that the proposed receiver achieves smaller BER than other suboptimum multiuser detectors and its performance approaches the single user bound. It is shown that the proposed receiver achieves a near-far resistance and increase of user capacity.

V. CONCLUSIONS

Interference suppression scheme presented here enables synchronous near-far resistant CDMA communication without the need for a power control. To mitigate the near-far effect, the DDFD is combined with the adaptive MMSE detector. The BER of the proposed receiver is evaluated by simulation and compared with those of other suboptimum multiuser detectors in a Rayleigh fading channel. It is shown that the proposed receiver with moderate computational complexity achieves superior BER performance than other detectors and approaches the single user bound. It is also shown that the proposed receiver achieves a near-far resistance and increase of user capacity.

REFERENCES


2204
Fig. 1. BER for the weakest user in a Rayleigh fading channel.

(a) fast fading, $f_s/T = 0.01$

(b) slow fading, $f_s/T = 0.001$

Fig. 2. BER for the weakest user in a fast fading channel at the average SNR = 20 dB.

Fig. 3. BER with the received power difference in a fast Rayleigh fading channel. Average SNR(1) = 20 dB and $i = 2, 3, \ldots, 10$.