Abstract - In this paper, multiuser detectors for a synchronized code division multiple access (CDMA) channel are adopted for detecting a spread spectrum signal in the presence of a digitally modulated narrowband interference (NBI). The bit error probability of a decorrelating detector is independent of the ratio of the power of a digitally modulated NBI signal to that of a spread spectrum signal but not independent of the time offset between two signals. A decorrelating decision feedback detector achieves the bit error probability which is independent of both the power ratio and the time offset between two signals in the regions of practical interest where the power of the NBI is greater than that of the spread spectrum signal. The bit error probability of a decorrelating decision feedback detector is lower than that of a decorrelating detector and coincides with an optimum single user bound in the regions of practical interest.

I. INTRODUCTION

It is well known that a direct sequence spread spectrum communication system has an inherent immunity to a narrowband interference (NBI) whether it is intentional or not. Recently, many studies have been done for the purpose of improving the NBI suppression capability further more [1-3]. However, since those studies have been based on very simple models for the NBI, i.e., single tone or autoregressive process, techniques based on these simple models are not effective when used for a digitally modulated NBI signal with much lower data rate than the spread spectrum chip rate. In such a frequency overlay situation, a new spread spectrum receiver is required which can effectively detect a spread spectrum signal while simultaneously suppressing the digitally modulated NBI by exploiting its characteristics.

Usually, a digitally modulated NBI signal has a higher power level than a spread spectrum signal. The significant difference in power level between these two signals may be regarded as the near-far effect which is a serious obstacle in a direct sequence code division multiple access (DS/CDMA) system. While a recent DS/CDMA system has been successful due to a stringent power control, a lot of attention has been directed to the area of multiuser detection [4-7]. Multiuser detection techniques have the capability to mitigate the near-far effect and, hence, can relax the dependence on a stringent power control. The near-far resistance of some multiuser detectors has led multiuser detection techniques to be applied to detecting the spread spectrum signal in the presence of the digitally modulated NBI [8].

In general, each bit of a spread spectrum signal is not synchronized with that of a digitally modulated NBI. Because of the asynchronous relation between these two signals, the problem under consideration may be modeled as the multiuser detection in an asynchronous code division multiple access (CDMA) channel. Multiuser detection, however, is more complex in an asynchronous CDMA channel than in a synchronous CDMA channel. Hence, to alleviate the added complexity caused by the asynchronous relation between signals, a one-shot approach has been taken in this paper, i.e., each bit interval of the spread spectrum signal is considered separately.

The rest of this paper is organized as follows. Section II contains the description of a received signal model and formulates the problem under consideration into the multiuser detection in a synchronous CDMA channel. In Section III, a decorrelating detector and a decorrelating decision feedback detector are introduced. In Section IV, performance analysis is carried out in terms of the bit error probability for the detectors in Section III. In Section V, numerical results are presented and, finally, some conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a system where a spread spectrum signal is transmitted over the additive white Gaussian noise (AWGN) channel in the presence of a digitally modulated NBI. The spread spectrum signal uses binary phase shift keying (BPSK) modulation scheme for data modulation, where each
information bit $b_i$ with time duration $T_i$ may take a value of $+1$ or $-1$ with equal probability. The signature waveform $p_{\tau_i}(t)$ of the spread spectrum signal is zero outside $[0,T_i)$ and normalized, i.e., $\int_{0}^{T_i} p_{\tau_i}(t) \, dt = 1$. The digitally modulated NBI signal uses BPSK modulation scheme for data modulation, where each information bit $d_i$, with time duration $T$, may take a value of $+1$ or $-1$ with equal probability. The pulse waveform $p_{\tau_i}(t)$ of the NBI signal is zero outside $[0,T)$ and normalized, i.e., $\int_{0}^{T} p_{\tau_i}(t) \, dt = 1$.

The baseband received signal at a spread spectrum receiver is given by

$$r(t) = \sum_{i=-\infty}^{\infty} \sqrt{P_{ss}} b_i p_{\tau_i}(t - i T_i - \tau_i) + \sum_{i=-\infty}^{\infty} \sqrt{P_{ss}} d_i p_{\tau_i}(t - i T_i - \tau_i) + n(t)$$

where the first, the second, and the third term on the right hand side are a spread spectrum signal, a digitally modulated NBI signal, and the AWGN with zero mean and power spectral density $\sigma_n^2$, respectively. $\tau_i$ and $\tau_i$ account for the time offset between the spread spectrum signal and the NBI signal. $P_{ss}$ and $P_{ss}$ represent the received signal power of the spread spectrum signal and the NBI signal, respectively. For a positive integer $K$, it is assumed that $T_i = KT_i$ and, without loss of generality, it is assumed that $\tau_i = 0$ and $0 < \tau_i < T_i$. Also, it is assumed that $\tau_i$ is time-invariant and known by the receiver.

Since each bit of the NBI signal can be regarded as a signal from a virtual spread spectrum user, it may be thought of that there exist $K + 1$ virtual spread spectrum users and a one true spread spectrum user within $[0,T_i)$. Let the normalized signature waveforms of the virtual spread spectrum users be defined as follows

$$s_{k,i}(t) = \begin{cases} \frac{1}{\sqrt{\alpha}} p_{\tau_i}(t - (K-1)T_i - \tau_i), & 0 \leq t < (K-1)T_i + \tau_i, \\ 0, & (K-1)T_i + \tau_i \leq t \leq T_i, \end{cases}$$

and

$$s_{k,i}(t) = \begin{cases} 0, & 0 \leq t < \tau_i, \\ p_{\tau_i}(t - \tau_i), & \tau_i \leq t < T_i + \tau_i, \\ 0, & T_i + \tau_i \leq t \leq T_i, \end{cases}$$

$$s_{K+1,i}(t) = \begin{cases} \frac{1}{\sqrt{1-\alpha}} p_{\tau_i}(t - (K-1)T_i - \tau_i), & (K-1)T_i + \tau_i \leq t \leq T_i, \end{cases}$$

where $\alpha = \int_{0}^{T_i} p_{\tau_i}^2(t + T - \tau_i) \, dt$. Then, (1) may be rewritten as

$$r(t) = \sum_{i=-\infty}^{K+1} \sqrt{w_i} s_{k,i}(t) + n(t) \quad \text{for } 0 \leq t < T_i$$

where $s_{k,i}(t) = p_{\tau_i}(t), \quad w_i = P_{ss}, \quad i = 1, 2, \ldots, K + 1, \quad w_{K+2} = P_{ss}, \quad \beta_i = \beta_d, \quad \beta_{K+1} = \beta_{K+1},$ and $\beta_{K+2} = b_{b}.$

Let the crosscorrelation matrix between signature waveforms be denoted by a $(K+2) \times (K+2)$ matrix $R$ with its $i$th row, $j$th column entry $R_{i,j} = \int_{0}^{T_i} s_{k,i}(t)s_{k,j}(t) \, dt$, $i,j = 1,2,\ldots, K+2$. Also, let a $(K+2) \times (K+2)$ diagonal matrix with $W_i = \sqrt{w_i}, \quad i = 1,2,\ldots, K+2$, be denoted by $W$. The sampled output vector of a bank of matched filters, each matched to the signature waveform of a corresponding user, is given by

$$y = RW\beta + \eta$$

where $y = [y_1, y_2, \ldots, y_{K+2}]^T$ with $y_i = \int_{0}^{T_i} r(t)s_{k,i}(t) \, dt$, $i = 1, 2, \ldots, K+2$, $\beta = [\beta_1, \beta_2, \ldots, \beta_{K+2}]^T$, and $\eta$ is a colored Gaussian noise vector with $\eta_i = \int_{0}^{T_i} n(t)s_{k,i}(t) \, dt$, $i = 1, 2, \ldots, K+2$, and the autocorrelation matrix $\Sigma^2 R$.

III. DETECTORS

A. Decorrelating Detector

If the sampled output vector $y$ of a bank of matched filters is applied to the matrix filter $R^{-1}$, the resulting output vector is given by

$$\tilde{y} = R^{-1}y = W\beta + z$$

where $z$ is a colored Gaussian noise vector with the autocorrelation matrix $\Sigma^2 R^{-1}$. The decision for the transmitted bit by the true spread spectrum user is given by

$$\hat{\beta}_{K+2} = \text{sgn}(\tilde{y}_{K+2}) = \text{sgn}(\sqrt{w_{K+2}} \beta_{K+2} + z_{K+2}).$$

$$\text{sgn}(x) = \begin{cases} +1, & x \geq 0 \\ -1, & x < 0 \end{cases}$$

2183
The decorrelating detector is optimum when there is no noise, i.e., $\eta = 0$. In practice, however, the decorrelating detector does not result in optimum performance due to the noise enhancement. Although the decorrelating detector is not optimum, it is the merit of the decorrelating detector that it does not need to know the interferer’s power and, also, its performance is independent of the interferer’s power.

B. Decorrelating Decision Feedback Detector

The positive definite crosscorrelation matrix $R$ can be factored as $R = F^T F$ by the Cholesky decomposition algorithm where $F$ is a lower triangular matrix [9]. If the sampled output vector $y$ of a bank of matched filters is applied to the matrix filter $(F^T)^{-1}$, the resulting output vector is given by

$$
\hat{y} = (F^T)^{-1} y = FW \beta + n
$$

where $n$ is a white Gaussian noise vector with the autocorrelation matrix $\sigma_n^2 I_{K+1}$. The decision for the transmitted bit by the true spread spectrum user is given by

$$
\hat{\beta}_{K+1} = \text{sgn}(\hat{y}_{K+1} + \sum_{i=1}^{K+1} F_{K+1, i} \sqrt{w_i} \hat{\beta}_i)
$$

where $F_{K+1, i} = 1$ and $\hat{\beta}_i$, $i = 1, 2, \ldots, K+1$, are the decisions for the virtual spread spectrum users which are obtained in a recursive manner.

IV. PERFORMANCE ANALYSIS

A. Decorrelating Detector

From (5), the bit error probability for the true spread spectrum user is given by

$$
P_e^{\text{DEC}} = Q(\sqrt{W_{K+2}} / \sigma_n^2 (R^{-1})_{K+2, K+2})
$$

Although the decorrelating detector completely eliminates the crosscorrelations between users, the noise power at the decorrelator output is enhanced to $\sigma_n^2 (R^{-1})_{K+2, K+2}$ which is greater than the noise power $\sigma_n^2$ at the output of the matched filter. Due to this noise enhancement, the decorrelating detector does not result in optimum performance and its performance degrades as the crosscorrelations between users increase.

B. Decorrelating Decision Feedback Detector

Provided that all the decisions for the virtual spread spectrum users are correct, the bit error probability for the true spread spectrum user is given from (7) as follows

$$
P_e^{\text{DEF}} = Q(\sqrt{W_{K+2}} / \sigma_n^2)
$$

which is a single user bound in the absence of any interference.

The exact bit error probability can be obtained by averaging the conditional error probability given a particular error pattern for users $1, 2, \ldots, K+1$ over all such error patterns

$$
P_e^{\text{DEF}} = E_{\Delta \beta} \left[ Q(\sqrt{W_{K+2}} / \sigma_n^2) + \sum_{i=1}^{K+1} F_{K+2, i} \sqrt{W_i} \Delta \beta_i / \sigma_n^2 \right]
$$

where $\Delta \beta_i = \beta_i - \hat{\beta}_i$, $i = 1, 2, \ldots, K+1$, and $E_{\Delta \beta}(\cdot)$ is the expectation over the ensemble of identical, uniformly distributed error patterns $\Delta \beta$. If all the previous decisions are correct, (10) becomes (9).

V. NUMERICAL RESULTS

An $m$-sequence of length 63, generated by the polynomial $g(x) = x^8 + x + 1$, is used as the spreading sequence. Rectangular pulses are used for a chip pulse of a spread spectrum signal and $p_c(t)$ of an NBI signal. Since larger values of $K$ render the computation of bit error probability very difficult and they also conflict with the assumption of NBI, we confine our consideration to two cases, $K = 1$ and $K = 2$.

Numerical results for $K = 1$ and $K = 2$ are shown in Fig. 1 and Fig. 2, respectively, when $P_{\text{NBI}} / \sigma_n^2$ is 11.4 dB. The bit error probability of a decorrelating detector is independent of the power ratio $P_{\text{NBI}} / P_{\text{ss}}$, but it is not independent of the time offset $\tau_1$ between two signals since $(R^{-1})_{K+2, K+2}$ in (8) depends on the time offset $\tau_1$.

In Fig. 1 and Fig. 2, the bit error probability of a decorrelating decision feedback detector is independent of both $P_{\text{NBI}} / P_{\text{ss}}$ and $\tau_1$ in the regions of practical interest where the power of an NBI is greater than that of a spread spectrum signal. The bit error probability of a decorrelating decision feed-

---

1. $I_r$ is an $r \times r$ identity matrix.
2. $Q(x) = 1 / \sqrt{2\pi} \int_{-\infty}^{x} e^{-t^2/2} dt$
back detector increases as $P_{sa}/P_{ss}$ decreases to the vicinity of 0 dB. This is due to the error propagation effect caused by the structure of an adopted decorrelating decision feedback detector. Since the effect of a digitally modulated NBI is negligible as $P_{sa}/P_{ss}$ decreases further, the bit error probability of a decorrelating decision feedback detector approaches a single user bound again. The decorrelating decision feedback detector achieves better performance than the decorrelating detector over all the range of $P_{sa}/P_{ss}$ and its bit error probability coincides with an optimum single user bound in the regions of practical interest where $P_{sa}/P_{ss}$ is high.

VI. CONCLUSIONS

In this paper, we have proposed the use of multiuser detectors for the detection of a spread spectrum signal in the presence of a digitally modulated NBI. To reduce complexity, proposed detectors use multiuser detection techniques developed for a synchronous CDMA channel. The bit error probability of a decorrelating detector is independent of the ratio of the power of a digitally modulated NBI to that of a spread spectrum signal $P_{sa}/P_{ss}$, but it is not independent of the time offset $\tau_s$ between two signals. The bit error probability of a decorrelating decision feedback detector, however, is independent of both $P_{sa}/P_{ss}$ and $\tau_s$ in the regions of practical interest where $P_{sa}/P_{ss}$ is high. A decorrelating decision feedback detector achieves a lower bit error probability than a decorrelating detector, and its bit error probability coincides with an optimum single user bound in the regions of practical interest.

REFERENCES

Fig. 1. Bit error probabilities when $K = 1$.

Fig. 2. Bit error probabilities when $K = 2$. 